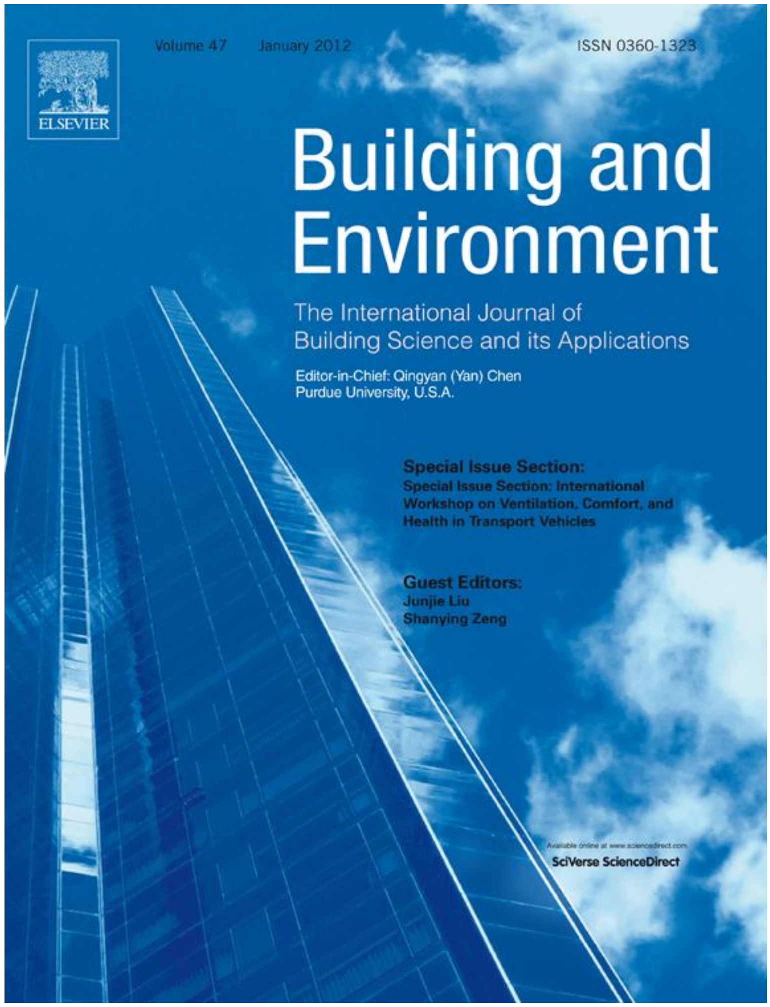
ELSEVIER
Volume 47 January 2012
ISSN 0360-1323
Building and Environment
The International Journal of Building Science and its Applications
Editor-in-Chief: Qingyan (Yan) Chen
Purdue University, U.S.A.
Special Issue Section:
Special Issue Section: International Workshop on Ventilation, Comfort, and Health in Transport Vehicles
Guest Editors:
Junjie Liu
Shanying Zeng
SciVerse ScienceDirect

Building and Environment 47 (2012) 373–384

Contents lists available at ScienceDirect

Building and Environment

journal homepage: www.elsevier.com/locate/buildenv

Reconsideration of parameter estimation and reliability evaluation methods for building airtightness measurement using fan pressurization

Hiroyasu Okuyama*, Yoshinori Onishi [1]

Institute of Technology, Shimizu Corporation, 4-17 Etchujima, 3-chome Koto-ku, Tokyo 135-8530, Japan

A R T I C L E I N F O

Article history:
Received 11 March 2011
Received in revised form
22 June 2011
Accepted 22 June 2011

Keywords:
Building airtightness
Testing method
Weighted least-squares
Discrepancy ratio
Uncertainty propagation
Reliability evaluation

A B S T R A C T

Building airtightness is among the most important performance indices of healthy indoor air quality, condensation, the building stack effect, and heating and cooling load caused by infiltration. Performance parameters are usually measured by testing methods involving pressurization or depressurization by means of a mechanical fan. Similar testing standards have now been established in ISO, ASTM, and JIS. All methods entail finding two parameters from some measurements of the inside and outside pressure difference and the airflow rate. Although these measurement data analysis methods are described in informative annexes, these are important techniques and have problems to be reconsidered and solved as a stochastic estimation and uncertainty evaluation. In the present paper, we examine improvement using weighted least-squares, correction of the parameter estimation equation, and deduction of the uncertainty propagation equation from not only the measurement uncertainty but also the residual of the model equation. Also, a reliability evaluation index capable of checking the appropriateness of the measurement is proposed. Through a computational experiment, the precision of the estimated parameters, the uncertainty of these parameters, and the reliability indices are investigated. Further, the present method is applied to actual measurement data and its practicality is also verified.

1. Introduction

Building airtightness is an important aspect of architectural performance which affects a wide range of phenomena. Examples include problems related to energy consumption due to heating and cooling infiltrating air, temperature-related comfort levels, health issues related to air quality, interior condensation of water vapor within walls, and building stack effects. Airtightness, however, does not always follow the design intention. It is also difficult to predict the location and extent of leaks from the appearance of a building. Methods of on-site building airtightness measurement are therefore important.

The airtightness evaluation model of a building envelope assumes that rates of the infiltration and exfiltration are proportional to the internal and external pressure difference with an exponent in the range of 0.5–1. The performance index of airtightness is calculated from this exponent n and a proportionality coefficient C. Determination of these two parameters is most commonly accomplished through experimental measurements, using a fan to pressurize or depressurize the building. Measurement standards are currently established according to standards such as ISO9972 [1], ASTM-E779-10 [2], and JIS-A2201 [3]. In each of these standards, a similar data analysis method is described, which is based on the measurement of several sets of airflow rates and the internal and external pressure differences to obtain the two parameters. Methods for analysis of such measurement data are described in an informative annex. Nonetheless, these are important techniques that have problems worthy of reconsideration and improvement in terms of a mathematical method of stochastic estimation and reliability evaluation.

First, the following can be offered as items related to measurement standards that are insufficient or problematic at present:

(a) *Simultaneous solutions of least-squares*
For the two parameters in the base modeling equation $q = C \cdot \Delta p^n$, there is a least-squares solution for the exponent n, but the coefficient C is solved using n and does not permit for simultaneous solution of both parameters.

(b) *Evaluation of premises established for measurement and regression modeling*
Conventional methods of reliability evaluation rely upon finding the confidence intervals of estimated parameters. In actuality, however, it is rare that the various conditions for measurement and regression (for example, conditions such as invariability in the airtightness parameters and small

* Corresponding author. Tel.: +81 3 3820 6438; fax: +81 3 3820 5955.
E-mail addresses: okuyama@shimz.co.jp (H. Okuyama), ohnishi@shimz.co.jp (Y. Onishi).
[1] Tel.: +81 3 3820 6439; fax: +81 3 3820 5955.

0360-1323/$ – see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.buildenv.2011.06.027

disturbance effects from wind pressure) hold, so the problem of negative effects on estimated parameters is a significant one. Evaluative indices of the need for further measurement and correction of measurement conditions are therefore of primary importance.

(c) *Weighting of measurement values (Δp_j,q_j)*
When making one of a number of measurements, it is always possible for sudden changes in external factors to occur or for prerequisites to measurement to fail, causing relatively large measurement errors. The least-squares method is susceptible to other negative effects on measurement values. It is therefore desirable to differentiate between the contributions to regression of the various measured values by applying varied weights, thus lessening such negative effects. It is even possible that in some extreme circumstances the application of some measurements to regression is not appropriate, so it is desirable to be able to reject such measurements.

(d) *A propagation equation for uncertainty variance in estimated parameters*
In conventional methods, residual and equation errors do not explicitly appear in the equations used to describe uncertainty variance for the two estimated parameters. Put another way, there are no coupled error propagation equations for the two estimated parameters that result from regression equation error. Further, there is no propagation equation for the estimated parameter uncertainty variance from the measurement uncertainty variance. It is therefore impossible to evaluate the effects on parameter estimations based on measurement uncertainty alone. Were it possible to take such effects into consideration, then measurements and the establishment of regression model prerequisites as described in (b) would be possible.

In this paper, we modify the parameter estimation equation to allow for a simultaneous least-squares solution, and furthermore make improvements to the weighted least-squares. In regard to the method of reliability evaluation, we first deduce the uncertainty propagation equation used to calculate the parameter estimation uncertainty variance. This uncertainty propagation is used for two types of uncertainty sources, one for causes of measurement uncertainty and one for causes of failure of the premises of the equation model. We also derive new reliability indices for evaluating the appropriateness of measurements and regression. Also, we use computational experimentation to investigate the accuracy of the estimated parameters, as well as the uncertainty of these parameters and the reliability indices. We furthermore apply the present method to actual measurement data to verify its practicality. Additionally, we consider the possibility of the basic model of the quadratic equation instead of the conventional power law equation.

2. The parameter estimation equations and reliability evaluation methods

2.1. Equation model and regression equation

The basic equation below is the same as that established for ISO and ASTM. As an aside, the JIS standard uses $1/n$ as the exponent instead of n. We shall determine the coefficient C and exponent n parameters through regression, using measurements of several internal/external pressure differentials Δp_j and fan pressurization flow rates q_j, and derive a theory for the evaluation of parameter reliability.

$$q_j = C \cdot \Delta p_j^n \tag{1}$$

We linearize Equation (1) by taking the logarithm of each side:

$$\log_e\left(q_j\right) = \log_e (C) + n \cdot \log_e \left(\Delta p_j\right) \tag{2}$$

We define the new variables x_i and y_i as follows:

$$x_j = \log_e\left(\Delta p_j\right) \tag{3}$$

$$y_j = \log_e\left(q_j\right) \tag{4}$$

In order to derive a method for finding the best-fitting estimations of $\log_e(C)$ and n, as well as to derive a method for finding their confidence intervals, we first define Equation (2) in matrix form based on the linearization of x_j and y_j, and rewrite as a regression equation:

$$y_j = [1\ x_j] \cdot \begin{bmatrix} \log_e (C) \\ n \end{bmatrix} = \mathbf{Z} \cdot \mathbf{a} \tag{5}$$

2.2. Parameter estimation equations using weighted least-squares

The equation error for application of the least-squares method is defined as the following vector matrix:

$$e_j = y_j - \mathbf{Z}_j \cdot \mathbf{a} \tag{6}$$

The sum of the squares of the equation errors for a group of np total measurements is defined as an evaluation function J according to the following equation. Here, w_j is the weight of each of the measured values, the calculation of which is described below. A t at the upper left of a variable indicates a transposition. The transposition of a scalar variable results in the original scalar, but there is nonetheless a reason for adding the transposition label t. Because the scalar value e_j contains the product of a matrix and a vector, in order to have the least-squares evaluation function result in a scalar value, it is necessary for e_j, which is multiplied from the left, to be a transposition.

$$\begin{aligned} J &= \sum_{j=1}^{np} {}^t e_j \cdot w_j \cdot e_j = \sum_{j=1}^{np} {}^t\left(y_j - \mathbf{Z}_j \cdot \mathbf{a}\right) \cdot w_j \cdot \left(y_j - \mathbf{Z}_j \cdot \mathbf{a}\right) \\ &= \sum_{j=1}^{np} \left({}^t y_j \cdot w_j \cdot y_j - {}^t y_j \cdot w_j \cdot \mathbf{Z}_j \cdot \mathbf{a} - {}^t\mathbf{a} \cdot {}^t\mathbf{Z}_j \cdot w_j \cdot y_j + {}^t\mathbf{a} \cdot {}^t\mathbf{Z} \cdot w_j \cdot \mathbf{Z}_j \cdot \mathbf{a}\right) \end{aligned} \tag{7}$$

To differentiate the evaluation function J, we next use the vector $\mathbf{a}$, which contains $\log_e(C)$ and n which we seek. Setting the differential to 0, the value $\mathbf{a}$ that satisfies the equation is the best estimated value.

$$\frac{\partial J}{\partial \mathbf{a}} = \sum_{j=1}^{np} \left(- {}^t\mathbf{Z}_j \cdot w_j \cdot y_j - {}^t\mathbf{Z}_j \cdot w_j \cdot y_j + 2 \cdot {}^t\mathbf{Z}_j \cdot w_j \cdot \mathbf{Z}_j \cdot \mathbf{a}\right) = 0 \tag{8}$$

This can be solved for the estimated parameter vector $\mathbf{a}$ by using the following equation:

$$\hat{\mathbf{a}} = \left[\sum_{j=1}^{np} \left({}^t\mathbf{Z}_j \cdot w_j \cdot \mathbf{Z}_j\right)\right]^{-1} \cdot \sum_{j=1}^{np} \left({}^t\mathbf{Z}_j \cdot w_j \cdot y_j\right) \tag{9}$$

Because this is a two-dimensional inverse matrix, not one of high degree, we can use the matrix elements to develop an explicit solution.

$$\hat{\mathbf{a}} = \begin{bmatrix} \log_e(C) \\ n \end{bmatrix} = \left(\sum_{j=1}^{np} {}^t\mathbf{Z}_j \cdot w_j \cdot \mathbf{Z}_j\right)^{-1} \cdot \left(\sum_{j=1}^{np} {}^t\mathbf{Z}_j \cdot w_j \cdot y_j\right) = \begin{bmatrix} \sum_{j=1}^{np} w_j & \sum_{j=1}^{np} w_j \cdot x_j \\ \sum_{j=1}^{np} w_j \cdot x_j & \sum_{j=1}^{np} w_j \cdot x_j^2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} \sum_{j=1}^{np} w_j \cdot y_j \\ \sum_{j=1}^{np} w_j \cdot x_j \cdot y_j \end{bmatrix}$$
$$= \frac{1}{\left(\sum_{j=1}^{np} w_j\right) \cdot \left(\sum_{j=1}^{np} w_j \cdot x_j^2\right) - \left(\sum_{j=1}^{np} w_j \cdot x_j\right)^2} \begin{bmatrix} \sum_{j=1}^{np} w_j \cdot x_j^2 & -\sum_{j=1}^{np} w_j \cdot x_j \\ -\sum_{j=1}^{np} w_j \cdot x_j & \sum_{j=1}^{np} w_j \end{bmatrix} \cdot \begin{bmatrix} \sum_{j=1}^{np} w_j \cdot y_j \\ \sum_{j=1}^{np} w_j \cdot x_j \cdot y_j \end{bmatrix} \tag{10}$$

We can also derive an expression to calculate the parameters in Equation (10):

$$\log_e(C) = \frac{\left(\sum_{j=1}^{np} w_j \cdot x_j^2\right) \cdot \left(\sum_{j=1}^{np} w_j \cdot y_j\right) - \left(\sum_{j=1}^{np} w_j \cdot x_j\right) \cdot \left(\sum_{j=1}^{np} w_j \cdot x_j \cdot y_j\right)}{\left(\sum_{j=1}^{np} w_j\right) \cdot \left(\sum_{j=1}^{np} w_j \cdot x_j^2\right) - \left(\sum_{j=1}^{np} w_j \cdot x_j\right)^2} \tag{11}$$

This allows us to solve for the coefficient C.

$$C = \exp\ (\log_e(C)) \tag{12}$$

The exponent n is calculated as follows:

$$n = \frac{\left(\sum_{j=1}^{np} w_j\right) \cdot \left(\sum_{j=1}^{np} w_j \cdot x_j \cdot y_j\right) - \left(\sum_{j=1}^{np} w_j \cdot x_j\right) \cdot \left(\sum_{j=1}^{np} w_j \cdot y_j\right)}{\left(\sum_{j=1}^{np} w_j\right) \cdot \left(\sum_{j=1}^{np} w_j \cdot x_j^2\right) - \left(\sum_{j=1}^{np} w_j \cdot x_j\right)^2} \tag{13}$$

2.3. *Coefficient of determination of the least-squares method*

As this estimation method is based on the least-squares method, one can calculate the coefficient of determination, which is the reliability evaluation index of the well-known multiple regression analysis. First, the residual of the regression equation is defined and calculated as follows:

$$v_j = y_j - \mathbf{Z}_j \cdot \hat{\mathbf{a}} \tag{14}$$

The sum of squares of the residual error is found by the following equation:

$$s(\hat{\mathbf{a}}) = \sum_{j=1}^{np} {}^t v_j \cdot w_j \cdot v_j = \sum_{j=1}^{np} w_j \cdot v_j^2 \tag{15}$$

The following equation gives the total variation:

$$s_y = \sum_{j=1}^{np} {}^t\left(y_j - \bar{y}\right) \cdot w_j \cdot \left(y_j - \bar{y}\right)$$
$$= \sum_{j=1}^{np} y_j \cdot w_j \cdot y_j - {}^t\left(\sum_{j=1}^{np} w_j \cdot y_j\right) \cdot \left(\sum_{j=1}^{np} w_j\right)^{-1} \cdot \left(\sum_{j=1}^{np} w_j \cdot y_j\right)$$
$$= \sum_{j=1}^{np} w_j \cdot y_j^2 - \left(\sum_{j=1}^{np} w_j\right)^{-1} \cdot \left(\sum_{j=1}^{np} w_j \cdot y_j\right)^2 \tag{16}$$

The sum of squares of the residual error and the total variation found in Equations (15) and (16), respectively, are then used to find the coefficient of determination (COD):

$$COD = 1 - \frac{s(\hat{\mathbf{a}})}{s_y} \tag{17}$$

2.4. Propagation equation for the uncertainty variance of the estimated parameters

We next derive the uncertainty variance of the estimated parameters. First, we use the following equation to denote the difference between the expected and estimated values of vector **a**:

$$\hat{\mathbf{a}} - E(\hat{\mathbf{a}}) = \left(\sum_{j=1}^{np} {}^t\mathbf{Z}_j \cdot w_j \cdot \mathbf{Z}_j\right)^{-1} \cdot \left(\sum_{j=1}^{np} {}^t\mathbf{Z}_j \cdot w_j \cdot y_j\right) - E\left\{\left(\sum_{j=1}^{np} {}^t\mathbf{Z}_j \cdot w_j \cdot \mathbf{Z}_j\right)^{-1} \cdot \left(\sum_{j=1}^{np} {}^t\mathbf{Z}_j \cdot w_j \cdot (\mathbf{Z}_j \cdot \mathbf{a} + e_j)\right)\right\} \tag{18}$$

From the vector in Equation (18), the expected value matrix will be created, as in the following equation. The matrix is the predicted parameter uncertainty variance-covariance matrix. We define the estimated uncertainty variance of $\log_e(C)$ and n, and the notation of covariance, as follows:

$$\mathbf{\Lambda}_a = E\{(\hat{\mathbf{a}} - E(\hat{\mathbf{a}})) \cdot {}^t(\hat{\mathbf{a}} - E(\hat{\mathbf{a}}))\} = \begin{bmatrix} \sigma^2_{\log C} & \sigma^2_{n \cdot \log C} \\ \sigma^2_{n \cdot \log C} & \sigma^2_n \end{bmatrix} = \left(\sum_{j=1}^{np} {}^t\mathbf{Z}_j \cdot w_j \cdot \mathbf{Z}_j\right)^{-1} \cdot \left(\sum_{j=1}^{np} {}^t\mathbf{Z}_j \cdot w_j \cdot E(e_j \cdot {}^t e_j) \cdot {}^t w_j \cdot \mathbf{Z}_j\right) \cdot {}^t\left\{\left(\sum_{j=1}^{np} {}^t\mathbf{Z}_j \cdot w_j \cdot \mathbf{Z}_j\right)^{-1}\right\}$$

$$= \frac{E(e_j \cdot {}^t e_j)}{\left\{\sum_{j=1}^{np} w_j \cdot \sum_{j=1}^{np} w_j \cdot x_j^2 - \left(\sum_{j=1}^{np} w_j \cdot x_j\right)^2\right\}^2} \cdot \begin{bmatrix} \sum_{j=1}^{np} w_j \cdot x_j^2 & -\sum_{j=1}^{np} w_j \cdot x_j \\ -\sum_{j=1}^{np} w_j \cdot x_j & \sum_{j=1}^{np} w_j \end{bmatrix} \cdot \begin{bmatrix} \sum_{j=1}^{np} w_j^2 & \sum_{j=1}^{np} w_j^2 \cdot x_j \\ \sum_{j=1}^{np} w_j^2 \cdot x_j & \sum_{j=1}^{np} w_j^2 \cdot x_j^2 \end{bmatrix} \cdot \begin{bmatrix} \sum_{j=1}^{np} w_j \cdot x_j^2 & -\sum_{j=1}^{np} w_j \cdot x_j \\ -\sum_{j=1}^{np} w_j \cdot x_j & \sum_{j=1}^{np} w_j \end{bmatrix} \tag{19}$$

The diagonal elements are the uncertainty variance of the parameters. The variance for $\log_e(C)$ is described using element (1,1) from Equation (19) as follows:

$$\sigma^2_{\log C} = E(e_j \cdot {}^t e_j) \cdot \left[\left(\sum_{j=1}^{np} w_j \cdot x_j^2\right)^2 \cdot \sum_{j=1}^{np} w_j^2 - 2 \cdot \left(\sum_{j=1}^{np} w_j \cdot x_j^2\right) \cdot \left(\sum_{j=1}^{np} w_j \cdot x_j\right) \cdot \left(\sum_{j=1}^{np} w_j^2 \cdot x_j\right) + \left(\sum_{j=1}^{np} w_j \cdot x_j\right)^2 \cdot \left(\sum_{j=1}^{np} w_j^2 \cdot x_j^2\right)\right] \Big/ \left[\sum_{j=1}^{np} w_j \cdot \sum_{j=1}^{np} w_j \cdot x_j^2 - \left(\sum_{j=1}^{np} w_j \cdot x_j\right)^2\right]^2 \tag{20}$$

From this, we calculate the uncertainty variance of the coefficient C as follows:

$$\sigma^2_C = \exp\left(\sigma_{\log C}\right)^2 \tag{21}$$

The variance for n is described using element (2,2) in Equation (19) as follows:

$$\sigma^2_n = E(e_j \cdot {}^t e_j) \cdot \left[\left(\sum_{j=1}^{np} w_j \cdot x_j\right)^2 \cdot \sum_{j=1}^{np} w_j^2 - 2 \cdot \left(\sum_{j=1}^{np} w_j\right) \cdot \left(\sum_{j=1}^{np} w_j \cdot x_j\right) \cdot \left(\sum_{j=1}^{np} w_j^2 \cdot x_j\right) + \left(\sum_{j=1}^{np} w_j\right)^2 \cdot \left(\sum_{j=1}^{np} w_j^2 \cdot x_j^2\right)\right] \Big/ \left[\sum_{j=1}^{np} w_j \cdot \sum_{j=1}^{np} w_j \cdot x_j^2 - \left(\sum_{j=1}^{np} w_j \cdot x_j\right)^2\right]^2 \tag{22}$$

2.5. Two methods for determining variance of equation error and weighting coefficients

Here we will consider two methods for defining $E(e_j \cdot {}^t e_j)$. One method is to take the average of the residual errors of the regression equations. Doing so takes into account not only the measurement error as a source of uncertainty, but also the effects of inadequate fulfillment of the premises for the regression equation model, such as invariability, uniformity, and linearity. The other method is one that assumes that only measurement uncertainty has an effect on the equation error. That method is described below.

When $E(e_j \cdot {}^t e_j)$ is taken from residual error, the calculation is performed using the following equation:

$${}_vE(e_j \cdot {}^t e_j) = \frac{1}{\sum_{j=1}^{np} w_j} \sum_{j=1}^{np} {}^t v_j \cdot w_j \cdot v_j = \frac{1}{\sum_{j=1}^{np} w_j} \sum_{j=1}^{np} v_j^2 \cdot w_j \tag{23}$$

Here, the v subscript to the left of the expected value function $E()$ indicates a residual error. This along with the uncertainty variance of the parameters calculated using Equations (20)–(22) are represented as ${}_v\sigma^2_C$ and ${}_v\sigma^2_n$, respectively.

Two methods of defining the weights w_j of each of the measured values can also be considered. One method is weighting by residuals, and the other method is weighting by measurement uncertainty.

In the first method, weighting by residuals, if a residual v_j is relatively large then it is taken to be a measurement value with low reliability, and is therefore assigned a relatively low weight. At first, all weights are set to 1 and the two parameters are estimated. After the residuals are calculated, the weights are calculated along the lines described, and then the parameters are estimated once again. This is repeated until the estimated parameters converge on some value.

In the weighting by residuals, a weight ${}_vw_j$ is calculated according to Tukey's biweight function [4]. During the repetition and convergence process, the following equation is used with the residual v_j of the previously estimated parameter and the expected value ${}_vE(e_j \cdot {}^te_j)$ of the equation error to calculate the next weight:

$$ {}_vw_j' = \left[1 - \left(\frac{1}{c_r^2}\right)\cdot\left(\frac{v_j^2}{{}_vE(e_j \cdot {}^te_j)}\right)\right]^2 \tag{24}$$

In cases where the calculation within the brackets on the right side of Equation (24) is negative, however, a weight of 0 is assigned. This process avoids negative effects on parameter estimation that would occur due to large errors caused by outlier measurements. In this equation, the constant c_r is generally taken to be some value between 5 and 9 [4]. It is possible, that in the case of measurement data of this sort such typical and conservative values need not be adhered to, with a smaller value being more appropriate. In the following case study, however, a minimum value of 5 is used.

In the second method, weighting by measurement uncertainty, for a given j-th measurement value the propagation rule from the measurement uncertainty variance to the equation error variance ${}_m\sigma_j^2$ is calculated according to the following equation:

$$\begin{aligned} {}_m\sigma_j^2 &= {}_m\sigma_q^2\cdot\left(\frac{\partial y_j}{\partial q_j}\right)^2 + {}_m\sigma_{\Delta p}^2\cdot\left(n\cdot\frac{\partial x_j}{\partial \Delta p_j}\right)^2 \\ &= {}_m\sigma_q^2\cdot\left(\frac{1}{q_j}\right)^2 + {}_m\sigma_{\Delta p}^2\cdot\left(\frac{n}{\Delta p_j}\right)^2 \end{aligned} \tag{25}$$

The measurement uncertainty variances are represented as ${}_m\sigma_q^2$ and ${}_m\sigma_{\Delta p}^2$ for the air flow measurement and the differential pressure measurement, respectively. The subscript m to the left of each variance means "measurement."

The expected value of the equation error variance from measurement uncertainty is calculated as the average of the variance of the measurement values in Equation (25). Here, the subscript m to the left of the expected value function $E()$ means "measurement."

$$ {}_mE(e_j \cdot {}^te_j) = \frac{1}{\sum_{j=1}^{np} w_j}\sum_{j=1}^{np} {}_m\sigma_j^2 \cdot w_j \tag{26}$$

Furthermore, in weighting by measurement uncertainty, too, a weight ${}_mw_j$ is calculated according to Tukey's biweight function.

$$ {}_mw_j' = \left[1 - \left(\frac{1}{c_r^2}\right)\cdot\left(\frac{{}_m\sigma_j^2}{{}_mE(e_j \cdot {}^te_j)}\right)\right]^2 \tag{27}$$

Once again, in cases where the calculation within the brackets on the right side of the equation is negative, a weight of 0 is assigned.

The parameters estimated with weighting by Equation (27) and the uncertainty variance of the parameters calculated using Equations (20)–(22) are represented as ${}_m\sigma_c^2$ and ${}_m\sigma_n^2$, respectively.

Weighting by residuals requires repeated calculations like those above until the calculations converge; in weighting by measurement uncertainty, too, because the exponent n is contained within the propagation rule for the error, repetition until convergence is required. The parameters C and n are also both estimated using these two weighting methods. The estimated standard deviations of their uncertainty are defined as ${}_v\sigma_C$ and ${}_v\sigma_n$, and ${}_m\sigma_C$ and ${}_m\sigma_n$, respectively. The calculation procedure is shown in Fig. 1.

It is likely better to utilize the results for parameters C and n by weighting residuals. The reason for this is that the cause of uncertainty in the estimated parameters is likely to arise from inadequate fulfillment of the premises for the regression equation model, such as invariability, uniformity, and linearity, rather than only the measurement uncertainty. Therefore, the uncertainty standard deviations of the estimated parameters should also be adopted by using the weighting from the equation residuals.

2.6. The discrepancy ratio β of the regression model premises and the estimated parameter standard deviations

The most important factor for actual implementation is an evaluation index allowing for determination as to whether the various premises for measurement and regression have been met, thereby allowing appropriate results to be obtained. This is related to item (b) in the list of problems given in Introduction. As shown in the following equations, the ratios of the standard deviations of estimation uncertainty for those parameters derived from the regression residuals ${}_v\sigma_C$ and ${}_v\sigma_n$, and the standard deviations of those derived from the measurement error ${}_m\sigma_C$ and ${}_m\sigma_n$ are taken, and defined as the discrepancy ratio β of the regression model premises [5,6]. If this ratio β is greater than 1, it is assumed that some of the premises were not upheld, and thus that either the measurement should be performed again or the premises for the regression model and measurements should be reconsidered.

$$\beta_C = \frac{{}_v\sigma_C}{{}_m\sigma_C} \tag{28}$$

$$\beta_n = \frac{{}_v\sigma_n}{{}_m\sigma_n} \tag{29}$$

2.7. Confidence interval of the estimated parameters

The computerization of measurement devices in recent years has simplified taking numerous measurements. While in the past it has been necessary to use the suppositions of a t-distribution to estimate confidence intervals for a limited number of measurements, it is now more commonly possible to use direct calculations and the postulates of the various variances and normal distributions to estimate the confidence intervals. In other words, in the case where Equations (20)–(22) are taken as the propagation equations from regression equation residual, it will be possible to estimate the confidence intervals of the estimated parameter from the standard deviations ${}_v\sigma_C$ and ${}_v\sigma_n$ and a required accuracy.

However, since least-squares regression is performed in the logarithmic space, the assumption of a normal distribution of the probability density is valid only in that logarithmic space and not in the real space. The probability of the estimated $\log_e(C)$ is distributed with the standard deviation $\sigma_{\log C}$ from Equation (20). For example, when a probability of 0.99 is required, the normal distribution function gives the multiplier for $\sigma_{\log C}$ as 2.57, and the confidence interval will be the range of $\pm 2.57\sigma_{\log C}$ around the estimated $\log_e(C)$. Thus, after calculation of the upper and lower limits in logarithmic space, by taking the exp() transformation to the real space, the real confidence interval can be obtained.

On the other hand, the exponent n has been estimated in the same manner as in the real space, and the uncertainty variance σ_n^2 is calculated by Equation (22). Thus, the confidence intervals around the estimated n is $\pm 2.57\sigma_n$.

3. Verification case study

Using a commonly used spreadsheet application, we created a worksheet based on the description above and performed a verification case study.

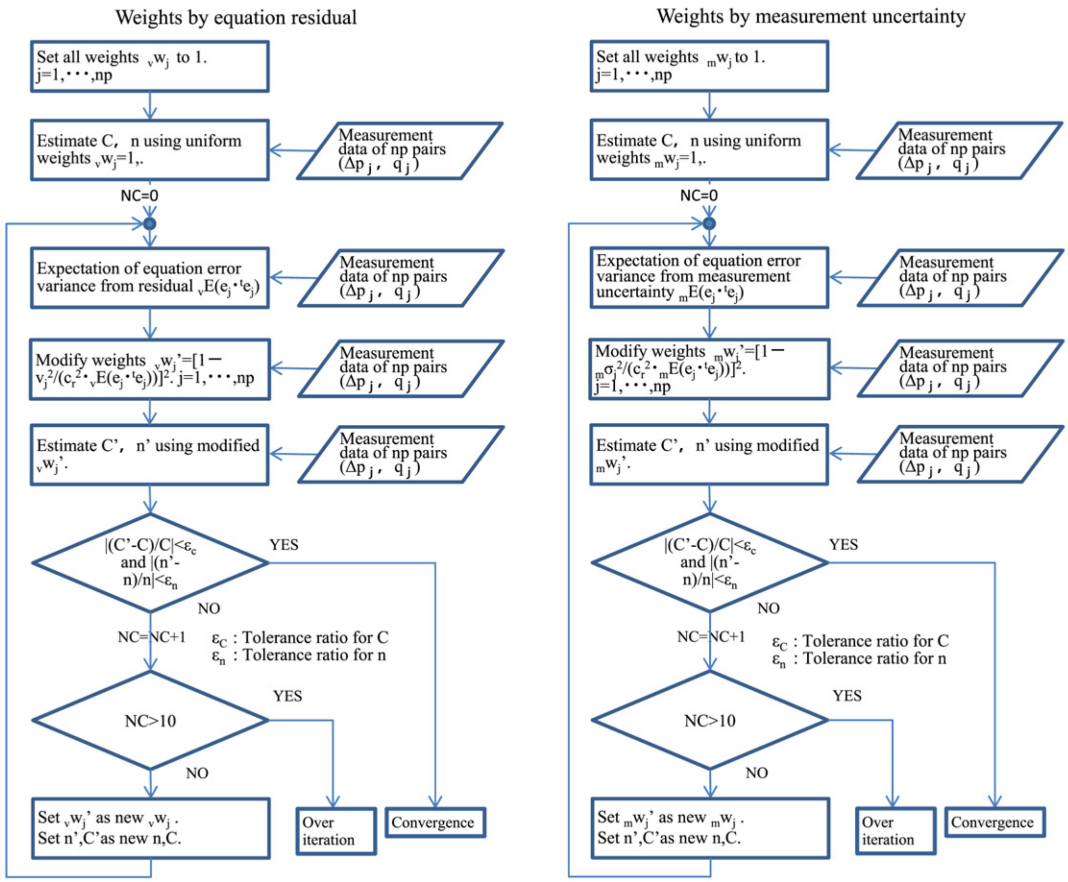

Fig. 1. Weighted least-squares calculation flow diagram using two types of weights.

3.1. Verification of estimation accuracy and reliability for estimated parameters C and n

We verified that the predicted standard deviations ${}_v\sigma_C$ and ${}_v\sigma_n$ for the estimated parameters C and n developed in this research would appropriately estimate the true error. To do so, we developed a model of an air flow rates network of a building with three airtightness grades and performed a computer simulated fan depressurization test. The simulation was performed using the thermal and air flow rates network simulation program NETS [7].

3.1.1. Finding the true values of C and n

Using the three-grade airtightness model, we conducted simulations of exhaust fan depressurization at a fixed airflow rate, and created 15 pairs of measurements ($\Delta p_j, q_j$) distributed between the range of 10 Pa–50 Pa for internal/external pressure differences.

We took the airflow (q_j) and the internal/external pressure difference (Δp_j) values as the true measurement value with zero measurement uncertainty. In the simulation model, pressure differential readings were taken at floor level, internal and external temperatures were set to be the same, and external wind force was taken to be zero. Using this data with zero measurement

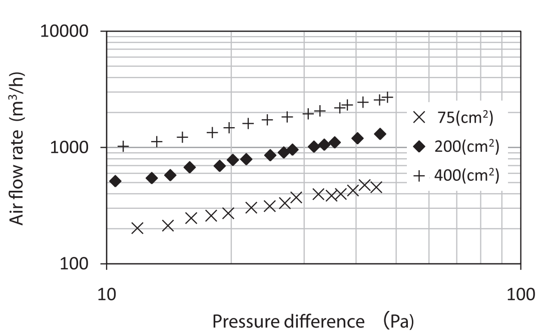

Fig. 2. Measurements of Δp_j and q_j by computational experiment.

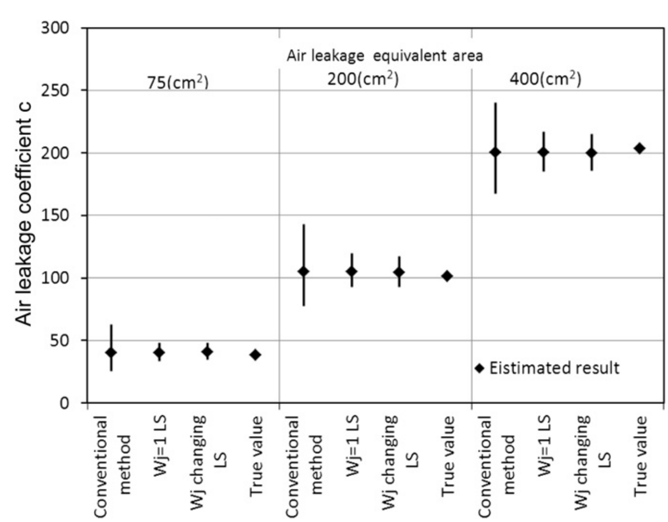

Fig. 3. Comparison of three estimation methods for C and confidence intervals.

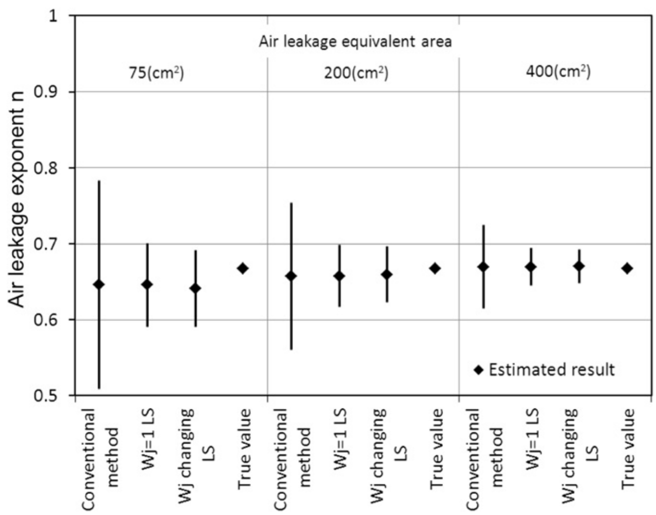

Fig. 4. Comparison of three estimation methods for n and confidence intervals.

uncertainty, we used the method for parameter estimation developed in the present study to estimate C and n, and took these as the true values.

3.1.2. Estimating C and n under the effects of measurement uncertainty

We next created simulated measurement values by using the proposed measurement uncertainty standard deviations ${}_m\sigma_{\Delta p}$ and ${}_m\sigma_q$ and a random number generator to create measurement errors, and applied these to the true values for the results of calculating Δp_j and the given air flows $q_j (j = 1,2,\ldots,np)$. In the ISO9972 standard, the uncertainty tolerance of the pressure difference measurement is specified to be ±2 Pa, so here ${}_m\sigma_{\Delta p}$ is assigned a value of 0.5 Pa (1/4 of that value). Also, ISO specifies the airflow rate uncertainty tolerance to be ±7% of the measurement range, so we supposed a full range of 1000 m^3/h and $1000 \times 0.07 \times (1/4) = 17.5$ to be approximated as ${}_m\sigma_q = 15$ (m^3/h).

We tested the three types of estimation and reliability evaluation methods: the conventional methods ISO, ASTM, and JIS; the method using a constant and uniform weighting coefficient of $w_j = 1$; and the method using a varying weighting coefficient of w_j. These latter two methods are described in the present paper. We performed a study on the basis of the aforementioned simulated measurement data. In the diagrams and tables, the least-squares method using constant weighting 1 is denoted as "$w_j = 1$ LS", and the method using changing weightings is denoted as "w_j changing LS".

3.1.3. Verification of predicted accuracy for C and n

We regarded the difference between the true values for C and n as described in Section 3.1.1 and the estimated parameters C and n with a measurement uncertainty effect as obtained in Section 3.1.2 as error. We compared the three types of predicted confidence intervals as described in Section 2.7: one is calculated by a conventional method (ISO, ASTM, or JIS) and the other two types are calculated from the standard deviations ${}_v\sigma_c$ and ${}_v\sigma_n$ from the regression residuals as developed in the present study. In this confidence interval calculation, the requested probability is assumed to be 0.99, the conventional methods employ a t-distribution, and the two proposed methods employ a normal distribution where the multiplier for the standard deviation is 2.57.

3.1.4. Comparison and discussion of prediction accuracy for C and n

Fig. 2 shows the simulated measurement data of buildings with three airtightness grades (equivalent leakage area: 75 cm^2, 200 cm^2, and 400 cm^2). Fig. 3 shows the true values of C, the estimated C, and the confidence intervals (requested probability: 0.99). Fig. 4 shows the same items for the exponent n. Table 1 lists these numerical values, along with the coefficient of determinant COD and the discrepancy ratio of model premises β, where the true value of exponent n is 1/1.5 for all cases.

- The estimation accuracy for parameters C and n is similar among the three methods. The weighted least-squares method is not highly effective in avoiding the unfavorable effect from the normal distribution measurement uncertainty. The weighted least-squares method will be useful for coping with sudden disturbances such as wind pressure change and will be discussed later.
- Confidence intervals estimated by the conventional methods are much wider than those estimated by the proposed method. In other words, the confidence interval accuracy of the conventional methods is lower than that of the present method. This same tendency is observed for the two parameters in all examined cases.
- COD is almost nearly equal to 1 for all cases. β exhibits a slightly larger change than COD. β_c and β_n are 1.0003 and 1.0054 respectively in the middle airtightness case, but in the other two cases, β is less than 1, thus indicating the appropriateness of the

Table 1
Results of numerical experiments using three methods to estimate parameters and reliability.

		Air leakage equivalent area 75 (cm^2)			Air leakage equivalent area 200 (cm^2)			Air leakage equivalent area 400 (cm^2)		
		Conventional method	$W_j = 1$ LS	W_j changing LS	Conventional method	$W_j = 1$ LS	W_j changing LS	Conventional method	$W_j = 1$ LS	W_j changing LS
Air leakage coefficient C	Estimated value	40.24	40.24	40.86	105.2	105.2	104.4	200.4	200.4	199.7
	βc	–	0.9624	0.9584	–	1.000	0.9964	–	0.9960	0.9941
	Upper limit of confidence interval	62.97	48.15	48.19	143.1	119.7	117.3	239.9	217.0	214.7
	Lower limit of confidence interval	25.72	33.64	34.65	77.29	92.41	92.82	167.3	185.0	185.8
	True value	37.98			101.6			203.1		
Air leakage exponent n	Estimated value	0.6461	0.6461	0.6408	0.6574	0.6574	0.6597	0.6696	0.6696	0.6707
	βn	–	0.6454	0.6016	–	1.005	0.9270	–	0.8856	0.8264
	Upper limit of confidence interval	0.7833	0.7010	0.6914	0.7543	0.6981	0.6965	0.7246	0.6940	0.6927
	Lower limit of confidence interval	0.5088	0.5911	0.5903	0.5604	0.6167	0.6228	0.6147	0.6453	0.6486
	True value	0.6668			0.6668			0.6668		
COD		0.9849	0.9849	0.9862	0.9919	0.9919	0.9927	0.9972	0.9972	0.9975

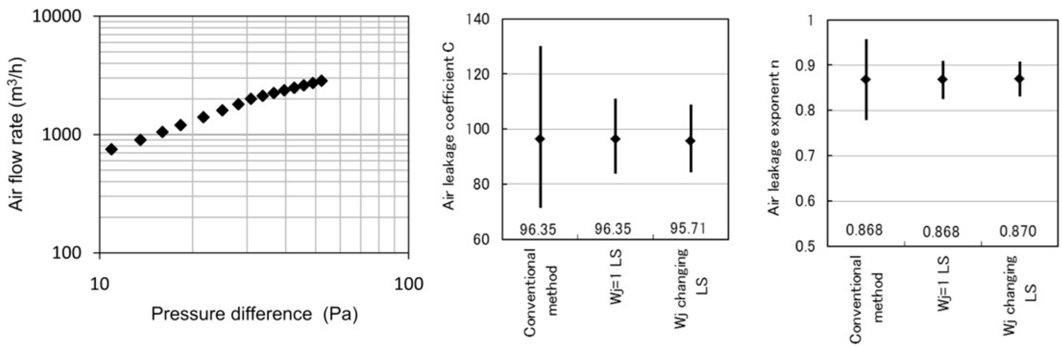

Fig. 5. Measurements and results by three estimation methods in case with varying equivalent air leakage area.

measurement. Under the condition where the assumed measurement uncertainty standard deviation for the evaluation is the same as the artificially generated uncertainty standard deviation, it is natural that the magnitude of β would be close to 1.

3.2. Evaluative capability when premise of invariability is not upheld

We created measurement values for simulated measurements of models with a changing leakage area, caused by internal/external pressure differentials. For example, the rubber valve leakage area will be affected by the pressure difference. Similar components and materials are also used in actual buildings.

In the testing model, the leakage exponent n is set to 1/1.5 and held constant, but the leakage area a (cm^2) changes according to the pressure difference Δp (Pa). For $0 \le \Delta p < 30$, $a = 250 + 5\Delta p$, and for $30 \le \Delta p$, $a = 400$. Simulated measurements were taken at 15 points within the range of 10–50 Pa. Where the equivalent leakage area a (cm^2) is defined by the following equation:

$$\Delta p = \frac{\rho}{2}\left(\frac{1}{0.36}\right)^{\frac{1}{n}} \cdot \left(\frac{q}{a}\right)^{\frac{1}{n}} \tag{30}$$

The data for the simulated measurement are shown by the left diagram in Fig. 5. The center and the right diagrams in Fig. 5 show the estimated values and the confidence intervals of C and n. Table 2 lists these numerical values, as well as COD and β, in the left half of the table.

In general, when making measurements, one is unable to recognize the change in the parameter C. However, it can be ascertained that the index β is greater than 1. Thus, we can determine whether the model premises are upheld. On the other hand, CODs are very close to 1 in any method, and problems with the model premises cannot be found from COD.

3.3. Capability of excluding data with sudden error due to disturbance

Tukey's biweight method can be used to eliminate outlying measurement values that are the result of sudden errors; accordingly, we investigated whether this function would work as expected. Assuming condition of a wind gust, we added error to 1 of the 15 measurements in the case with the equivalent leakage area of 400 (cm^2) described in Section 3.1. The pressure of the gust of wind is assumed 12 (Pa). The left diagram in Fig. 6 shows the measurement data distribution. The center and the right diagrams in Fig. 6 show the true values and the confidence intervals of estimated C and n. Table 2 lists these numerical values, as well as COD and β, in the right half of the table.

The biweight method assigned zero weight to that pair of data affected by the sudden disturbance. Good agreement is observed between the estimated parameters and the true values for only the results by the biweight method. In the constant weight method, β_n is much greater than 1, but in the varying weight method, β is less than 1. We can find the suddenly disturbed data by examining the weights and β in this way.

3.4. Verification using actual measurement data

We investigated whether the method of analysis and evaluation presented in this paper gave the desired results for actual building

Table 2
Comparison of parameter and reliability estimation by three methods for two cases with varying C and sudden wind disturbance.

		Case with varying C			Case with sudden disturbance		
		Conventional method	$W_j = 1$ LS	W_j changing LS	Conventional method	$W_j = 1$ LS	W_j changing LS
Air leakage coefficient C	Estimated value	96.35	96.35	95.71	241.7	221.5	199.4
	βc	–	1.014	1.011	–	1.085	0.9959
	Upper limit of confidence interval	130.0	110.9	108.9	558.2	296.8	214.0
	Lower limit of confidence interval	71.39	83.68	84.15	104.6	165.4	185.8
	True value	–			203.1		
Air leakage exponent n	Estimated value	0.8677	0.8677	0.8696	0.6059	0.6344	0.6717
	βn	–	1.339	1.285	–	3.489	0.8776
	Upper limit of confidence interval	0.9569	0.9096	0.9079	0.8584	0.7229	0.6932
	Lower limit of confidence interval	0.7786	0.8258	0.8313	0.3533	0.5458	0.6501
	True value	–			0.6668		
COD		0.9951	0.9951	0.9957	0.9951	0.9951	0.9957

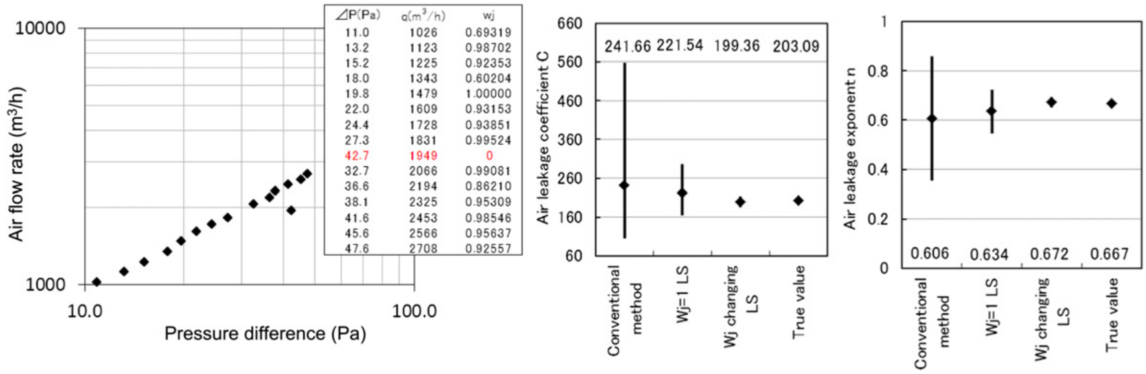

Fig. 6. Measurements and results for three estimation methods in case with sudden wind disturbance.

measurements. We took up five test cases for five different buildings. These cases have several different features such as airtightness, temperature difference inside and outside, and strong or weak wind conditions. These buildings are described below.

(1) Two-story log cabin. The temperature difference between inside and outside was large. Total floor area was 149.81 m^2. The measurement time was September 9, 2003, 13:45–15:00. Indoor and outdoor temperatures were 25.1 °C and 32.6 °C. Wind velocity was less than 1.0 m/s.
(2) Two-story steel-frame house. The climate was calm and there was no temperature difference between inside and outside. Total floor area was 114.27 m^2. The measurement time was December 17, 1999, 10:30–11:00. Indoor and outdoor temperatures were 12.5 °C and 12.5 °C. Wind velocity was less than 0.5 m/s.
(3) Two-story house with prefabricated thermal insulation panel. There was a slight breeze and no temperature difference between inside and outside. Total floor area was 95.06 m^2. The measurement time was November 11, 1999, 10:00–11:00. Indoor and outdoor temperatures were 15.0 °C and 15.0 °C Wind velocity was less than 1.0 m/s.
(4) Two-story lightweight steel-frame house. This structure was found to have low airtightness 5.0 cm^2/m^2 according to the JIS performance index. Total floor area was 84.47 m^2. The measurement time was November 5, 1999, 12:00–13:00. Indoor and outdoor temperatures were 19.3 °C and 18.5 °C. Wind velocity was less than 1.0 m/s.
(5) Two-story wooden-panel house. Wind speed was relatively high. Total floor area was 147.50 m^2. The measurement time was September 9, 1999, 10:30–11:30. Indoor and outdoor temperatures were 27.6 °C and 28.3 °C. Wind velocity was 1.0–3.0 m/s.

Fig. 7 shows the distribution of airflow rate and pressure difference measurements. Fig. 8 shows the estimated C and the confidence intervals making a comparison between the three methods. Similarly, the results for the estimated exponent n are shown in Fig. 9. Table 3 shows these numerical values along with COD and β.

We can see that the confidence intervals obtained by the conventional method are larger than that of the proposed method, and therefore we can describe the uncertainty of the conventional reliability evaluation method as being relatively large. In case (1) with a large temperature difference, as well as in case (5) with a large wind velocity, it is found that the discrepancy ratio β is greater than 1. Therefore, we can identify that the premises of the system identification model are not sufficiently fulfilled in these two cases. In spite of the undesirable results that β was greater than 1, COD of these two cases are nearly equal to 1; thus, COD is insufficient in the detection of this type of problem.

4. Scope of the index β and reconsideration of the conventional model

The discrepancy ratio β, as an indicator of fulfillment of measurement and system identification premises, is generally not effective in relation to systematic uncertainty such as measurement device calibration errors or defects. This is also a limitation on using the weighted least-squares method to reduce the miscellaneous negative effects of disturbances.

However, when using an insufficiently zero-adjusted differential pressure-measuring instrument, for example, there is a possibility of a non-zero airflow rate even if the instrument reads zero. Such a case may result in structural differences between the regression equation and the real-world phenomenon, increasing the residue and therefore making the discrepancy ratio β become greater than 1. In this way, we can suppose insufficient realization of some premises. Namely, if the real-world phenomenon differs from the regression equation structure, or if parameters that should remain constant are varying, then in many cases these differences will appear as regression equation residuals and we can suppose insufficient realization of the premises based on the index β. Such an evaluation, however, is only appropriate when all weightings of w_j are 1.

In the actual building measurement case (1) in Section 3.4, which featured a relatively large buoyancy effect caused by internal and external temperature differences, β_n is 1.467 and β_C is 1.024. In case (5), which had a relatively large wind pressure, β_n is 1.078 and β_C is 1.004. In the case of a varying opening area in Section 3.2, β_n is 1.339 and β_C is 1.014. In Section 3.3 case with gusting winds, β_n is 3.489 and β_C is 1.085. This case of disturbance due to wind gusts causes a measurement value to be an outlier from the group, and the weight w_j becomes zero and the measurement will be excluded.

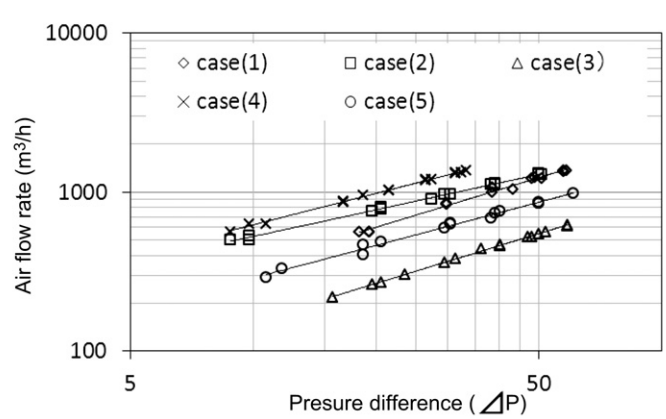

Fig. 7. Five cases of measurement data for actual buildings.

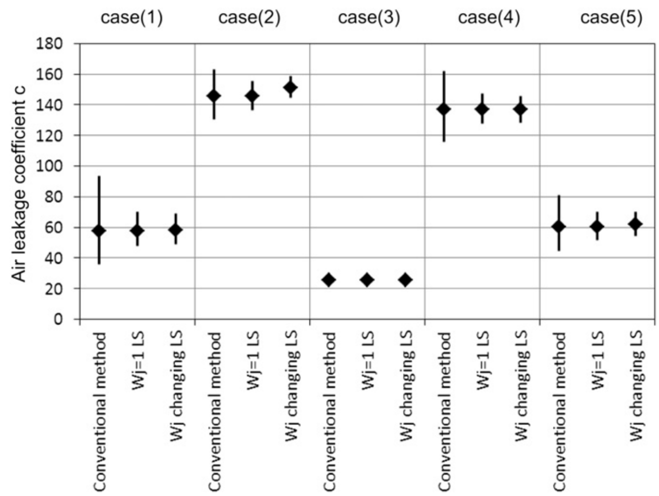

Fig. 8. Comparison of C estimated by three methods for five actual building cases.

Section 3.1 shows an idealized case with only normally distributed measurement uncertainty. In this case, the standard deviation of the assumed measurement uncertainty and that of the artificial error generation from random numbers are same, and thus β is less than or approximately 1.

The basic Equation (1) can be also reconsidered. Etheridge and Sandberg [8] introduced the idea that the quadratic equation is preferable to the power law equation of (1). The quadratic equation for pressure loss consists of the sum of the two terms q^2 and q. If we consider the power law equation as an engineering-based model, then the quadratic equation may be considered a physics-based model. The term q^2 represents kinetic energy dissipation by inertial resistance, and the term q represents friction pressure loss by viscosity. By referring to equations (3.8.6), (3.8.7) and (3.8.8) in Etheridge and Sandberg [8], we can define the following quadratic equation. For an air density ρ and a viscosity μ,

$$\begin{aligned}\Delta p_j &= d_i \cdot \frac{\rho}{2} \cdot \left(\frac{1}{0.36}\right)^2 \cdot \left(\frac{q_j}{a}\right)^2 + d_v \cdot \mu \cdot \left(\frac{2 \times 100}{g_w}\right) \cdot \left(\frac{1}{0.36}\right) \cdot \left(\frac{q_j}{a}\right) \\ &= \left[\left(\frac{d_i \cdot \rho}{2 \times a^2}\right) \cdot \left(\frac{1}{0.36}\right)^2\right] \cdot q_j^2 + \left[\left(\frac{200 \times d_v \cdot \mu}{g_w \cdot a}\right) \cdot \left(\frac{1}{0.36}\right)\right] \cdot q_j \\ &= f_i \cdot q_j^2 + f_v \cdot q_j\end{aligned} \tag{31}$$

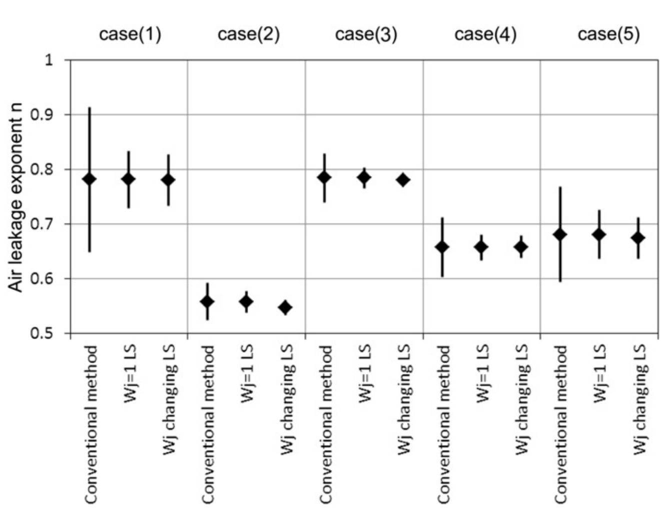

Fig. 9. Comparison of n estimated by three methods for five actual building case.

Table 3
Results of actual building tests using three methods to estimate parameters and reliability.

		Case (1)			Case (2)			Case (3)			Case (4)			Case (5)		
		Conventional method	$W_j = 1$ LS	W_j changing LS	Conventional method	$W_j = 1$ LS	W_j changing LS	Conventional method	$W_j = 1$ LS	W_j changing LS	Conventional method	$W_j = 1$ LS	W_j changing LS	Conventional method	$W_j = 1$ LS	W_j changing LS
Air leakage coefficient C	Estimated value	57.86	57.86	58.00	145.7	145.7	151.4	25.46	25.46	25.72	137.0	137.0	136.8	60.07	60.07	61.69
	βc	–	1.024	1.017	–	0.9894	0.9837	–	0.9322	0.9238	–	0.9810	0.9792	–	1.004	0.9975
	Upper limit of confidence interval	93.51	69.98	68.80	163.0	155.5	158.6	29.95	27.20	26.70	161.8	147.2	145.6	80.97	70.05	70.35
	Lower limit of confidence interval	35.79	47.83	48.90	130.2	136.5	144.5	21.65	23.84	24.77	116.0	127.6	128.4	44.56	51.51	54.10
Air leakage exponent n	Estimated value	0.7814	0.7814	0.7804	0.5577	0.5577	0.5471	0.7844	0.7844	0.7811	0.6575	0.6575	0.6581	0.6812	0.6812	0.6742
	βn	–	1.467	1.350	–	0.7045	0.5160	–	0.2677	0.1551	–	0.5916	0.5380	–	1.078	0.9504
	Upper limit of confidence interval	0.9141	0.8340	0.8276	0.5918.	0.5775	0.5610	0.8296	0.8028	0.7915	0.7121	0.6809	0.6787	0.7684	0.7260	0.7124
	Lower limit of confidence interval	0.6487	0.7288	0.7332	0.5236	0.5380	0.5332	0.7392	0.7660	0.7707	0.6030	0.6341	0.6375	0.5940	0.6363	0.6359
COD		0.9905	0.9905	0.9920	0.9974	0.9974	0.9987	0.9988	0.9988	0.9996	0.9973	0.9973	0.9976	0.9909	0.9909	0.9927

Here, we call d_i the inertia discharge coefficient, d_v the viscosity discharge coefficient and g_w equivalent gap width (cm). In the original equations, d_i and d_v are explained using several parameters; nevertheless, deductive calculation might be difficult. We further call f_i and f_v the inertia regression coefficient and the viscosity regression coefficient, respectively. One method to obtain the equivalent leakage area a (cm^2) and the equivalent gap width g_w (cm) is to solve for the parameters a and g_w after regression for f_i and f_v and setting d_i and d_v to 1.

It is possible to develop similar least-squares system parameter estimations and uncertainty evaluation methods by regarding the quadratic Equation (31) as a new regression equation. A probable merit of the quadratic equation model of measurement data analysis is that we can expect better regression accuracy than with the conventional model, because the regression calculation of the conventional equation model is done in logarithmic space. Concerning the notation of the building airtightness performance index used in the JIS standard, the exponent n is not indicated but only a, which could be a problem. Also, even in the quadratic equation model g_w would also be required, not only a.

5. Conclusions

We derived a weighted least-squares method to estimate the two parameters C and n from the measurement of Δp and q, and a method for evaluating the reliability of the estimated parameters. For the reliability evaluation, we deduced uncertainty propagation equations for the two parameters from the regression equation error, and defined the discrepancy ratio β for the premises of the system identification model.

We derived two methods for assigning weights to each of the measurement values: weighting by residuals and weighting by measurement uncertainty. Weighting by residuals is effective in reducing the undesirable effects caused by sudden disturbances such as a gust of wind. Weighting by measurement uncertainty is useful in that it assigns smaller weights to small pressure differentials and airflow measurement values for which uncertainty can be expected to be large. In most cases, however, one can assume that weighting by residuals is the more appropriate approach.

The introduced uncertainty standard deviation for the parameters allows one to know the confidence intervals within which true values can be found at a specified degree of certainty. The present method estimates the intervals more precisely than the conventional method: namely, the estimated interval around the true value is wider for the conventional method than the proposed method.

The discrepancy ratio β for the measurement and regression premises can be used in the determination of whether measurements and regression were a success or failure. The advantages of β for distinguishing the invariability of the estimated parameters were verified through a numerical experiment using a model in which the leakage area was varied by the pressure difference, and in this case β became greater than 1. We also confirmed the usefulness of the weighted least-squares method in reducing the undesirable effects of sudden disturbances, which is accomplished by assigning zero or small weight to measured values with large error. However the index β is generally not effective in relation to systematic uncertainty such as measurement device calibration errors.

When we tested the presented method using measurements of five actual buildings, the discrepancy ratio β became larger than 1 in the two cases where winds were strong or the temperature difference between inside and outside was great. This result shows that the discrepancy ratio β can be used to evaluate the establishment of premises for measurement and regression.

Furthermore, following a reconsideration of the basic model of the conventional power law equation, we can expect more reasonable and accurate performance indices with the quadratic equation model.

Acknowledgments

We thank Professor Hiroshi Yoshino, Chair of the Committee for Consideration of Revision of ISO 9972, for providing the opportunity and motivation for this research, as well as for his valuable advice. We also received useful related materials from Professor Yasuo Utsumi. We thank Mr. Katsuichi Kuroki of the Japan Testing Center for Construction Materials for providing us with measurement data from actual buildings. Finally, we thank the many members of the various relevant committees for valuable feedback.

Nomenclature

q_j	j-th air flow rate in np pairs of measurements (m^3/h)
Δp_j	j-th pressure difference in np pairs of measurements (Pa)
C	Parameter of leakage coefficient to be estimated
n	Parameter of leakage exponent to be estimated
x_j	$\log_e(\Delta p_j)$
y_j	$\log_e(q_j)$, y_j with top bar means average value
n_p	Total pairs of Δp_j and q_j measurements
$\mathbf{a}$	Vector containing parameters $\log_e(C)$ and n
$\hat{\mathbf{a}}$	Vector containing parameters $\log_e(C)$ and n estimated by the least-squares method
$\mathbf{Z}_j$	Row matrix containing values 1 and x_j
e_j	Regression equation error caused by j-th measurement
J	Evaluation function of least-squares
w_j	Weighting coefficient for j-th measurement
w'_j	Updated weighting coefficient in iterative convergence calculation
${}_v w_j$	Weighting coefficient for j-th measurement calculated from regression equation residual
${}_m w_j$	Weighting coefficient for j-th measurement calculated from measurement uncertainty
v_j	Regression equation residual for j-th measurement
$s(\hat{\mathbf{a}})$	Residual square sum
s_y	Total variation
COD	Coefficient of determination
$E(\,)$	Calculation operator of stochastic expectation
${}_v E(e_j \cdot {}^t e_j)$	Expected square of regression equation error from residual
${}_m E(e_j \cdot {}^t e_j)$	Expected square of regression equation error from measurement uncertainty
$\boldsymbol{\Lambda}_\mathbf{a}$	Variance and covariance matrix of estimated parameters
$\sigma^2_{\log\ C}$	Variance of estimated $\log_e(C)$ in logarithmic space
σ^2_C	Variance of estimated C
${}_v\sigma_C$	Standard deviation of estimated C from regression equation residual
${}_m\sigma_C$	Standard deviation of estimated C from measurement uncertainty
σ^2_n	Variance of estimated n
${}_v\sigma_n$	Standard deviation of estimated n from regression equation residual
${}_m\sigma_n$	Standard deviation of estimated n from measurement uncertainty
${}_m\sigma^2_{\Delta p}$	Measurement uncertainty variance of pressure difference
${}_m\sigma^2_q$	Measurement uncertainty variance of air flow rate
${}_m\sigma^2_j$	Regression equation error variance of j-th measurement
β_C	Discrepancy ratio of model premises for parameter C
β_n	Discrepancy ratio of model premises for parameter n
β	Generic notation of discrepancy ratio for all parameters

ρ Air density (kg/m^3)
a Equivalent leakage area (cm^2)
g_w Equivalent gap width (cm)
d_i Inertia discharge coefficient
d_v Viscosity discharge coefficient
μ Viscosity (Pa s)
f_i Inertia regression coefficient
f_v Viscosity regression coefficient

References

[1] ISO 9972. Thermal performance of buildings — determination of air permeability of buildings — fan pressurization method, Annex C (informative). Recommended procedure for estimating uncertainty in derived quantities. 2nd ed; 2006-05-01.

[2] ASTM E779-10. Standard test method for determining air leakage rate by fan pressurization, Annex A1 (mandatory information). Procedure for estimating precision errors in derived quantities; 2010-April.

[3] JIS A2201. Test method for performance of building airtightness by fan pressurization, Annex 3 (informative). Calculation of a, n and the confidence intervals. 1st ed; 2003-4-1.

[4] Nakagawa Toru, Koyanagi Yoshio. Experimental data analysis by least-squares method: computer program SALS. 1st ed. University of Tokyo Press.; 1982-05-30. pp. 163–176.

[5] Okuyama H. System identification theory of the thermal network model and an application for multi-chamber airflow measurement. Build Environ 1990;25(4): 349–63.

[6] Okuyama H, Onishi Y, Tanabe SI, Kashihara S. Statistical data analysis method for multi-zonal airflow measurement using multiple kinds of perfluorocarbon tracer gas. Build Environ 2009;44(3):546–57.

[7] Okuyama H. Thermal and airflow network simulation program NETS. Proc 6th Int IBPSA Conf (Building Simulation '99), Kyoto, September 1999; 1237–1244, [Notes of [7]: NETS is authorized by the Ministry of Land, Infrastructure and Transport Japan As a Calculation Method for Annual Heating and Cooling Loads of Buildings, 22 October 2002].

[8] Etheridge David, Sandberg Mats. Building ventilation theory and measurement. John Wiley & Sons Ltd, ISBN 047196087X; 1996. Sections 10.4.2 (Field measurements on building components and rooms) and 3.8.4 (Comparison of the quadratic and power law equations).