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STATE-SPACE APPROACH TO BUILDING ENVIRONMENTAL ANALYSIS USING THERMAL NETWORK CONCEPTS

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Synopsis

A theory for computer simulation of thermal performance and energy use in buildings is described. Heat transfer in buildings consists of multifarious forms. Thermal network concepts are described to be effective for automatically setting up a state equation in the state-space approach. Generalized heat conductance is defined for various heat transfer forms including mass flow and radiation. Concepts of network mode change are defined for the time varying parameter or non-linearity. Simulation is implemented by time integration of the state equation. For this scheme, an analytical solution based on the spectral resolution on eigenspaces rather than the approximated solution is reintroduced. A method for reducing the order of the state equation for computational efficiency is deduced simply from the weighted residual integration in the time domain. Also, the theory of coupling subsystems for systematic modeling is deduced from the direct sum of output equations from each system component.

Introduction

In the field of architectural environmental engineering, there are some characteristic problems in computerized prediction or analyses of thermal performance and energy use in buildings. First, there is multi-dimensional heat flow. This means, for example, that room temperature is affected by the surrounding variously positioned walls through inner transient heat conductions, and further multi-dimensional radiative heat transfers take place between wall surfaces. Second, there is heat flow which is caused by mass flow. For example, for air flows existing through the inner wall air layers, attic spaces, under floor space or heating and ventilating ducts, the problems include not only conduction but mass flow heat transfer as well. Third, there is the time varying parameter or non-linear problem. For example, if these air flow rates vary according to a certain schedule or temperature, they can be regarded as a thermal structure change.

These phenomena should be considered and accurately evaluated especially in passive solar houses. Buildings of this kind control thermal energy flow by natural means utilizing these phenomena. The thermal systems to be studied with such a nature can be written in a set of lumped-parameter, coupled differential algebraic equations from the fundamental physical laws of energy conservation. However, obtaining these parameters and a set of equations for the entire system by discretization methods, such as the finite element method, is not necessarily the most practical and efficient way in many cases. This is principally because the order of the set of equations becomes too large. Consequently, what is initially needed is a modeling method that can realize coupling and proper precision balance throughout the entire system without being hampered in small domains.

On the other hand, the state-space approach has been recognized in recent years to be an effective method of solving dynamic problems in many engineering fields. This method is based on the vector matrix ordinary differential state equation constructed by some number and order time derivative equations describing the dynamic system and defining proper state variables. Once the system is represented by this state equation, many powerful tools, such as linear algebra, modern control theory and vector matrix numerical methods, can be applied in its study. Therefore, what is needed secondly is automated and systematic methods for casting the thermal system problem into the state-

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space form.

The thermal network concepts presented in this paper represent the key to meeting these requirements and problems.

1. Formulation

Network concepts are based on the idea that any one part of the system is connected with all other parts of the system. Therefore, formulating the system according to these concepts implies that from one- to three-dimensional, and n-dimensional problems in the engineering field can be handled in this way. Consequently, this formulation can be called a perfect system formulation, because wide applications are perfectly realized from it.

These concepts involve the skeleton of the system structure and not the outer covering plates. One of the mathematical methods for determining the plates is the finite element method. In the field of architectural environmental engineering, however, these can be accomplished directly from the coefficient of heat transmission and the evaluating method in many cases.

The real phenomenon is modeled in n nodes with capacitances and generalized conductances among them. Here, generalized includes not only the meaning of conduction but also that of mass flow. At the arbitrary node of i, the equilibrium equation is then written as

$$m_{ii}\cdot\dot{x}_i=\sum_{j=1}^{n+n_0}c_{ij}\cdot x_j-\sum_{j=1}^{n+n_0}c_{ji}\cdot x_i+\sum_{j=1}^{ng}r_{ij}\cdot g_j \qquad (1)$$

where m_{ii} is thermal capacitance or volume of node i, x_i and x_j are temperatures of node i and j respectively, c_{ij} is generalized conductance from node j to i, g_j is the j-th generator of free input generators, r_{ij} is input ratio to node i from the j-th generator, n_0 is the total number of exogenous nodes, and n_g is the total number of free input generators. The dot denotes the time derivative.

Many of the other papers working in this area are inadequate in their formulation of boundary conditions (see Ref. 1). In order to construct the state equation of whole systems, formulation of Eq. (1) is considered to be the most rational. Therefore, defining $x={}^t(x_1, \cdots, x_n)$ as the state vector, $x_0={}^t(x_{n+1}, \cdots, x_{n+n0})$ as the fixed input vector and $g={}^t(g_1, \cdots, g_{ng})$ as the free input vector, the state equation (2) can be directly constructed from Eq. (1). Combining the second and third terms in the right side of Eq. (2), f in Eq. (3) can be called the heat flow input vector, such that

$$M\cdot\dot{x}=C\cdot x+C_0\cdot x_0+R\cdot g \qquad (2)$$

and

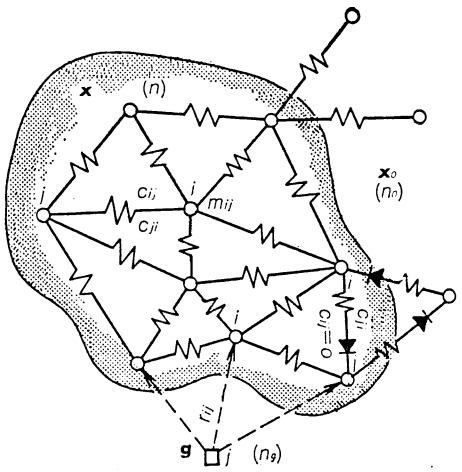

Fig. 1 Concepts of thermal network

$$f=C_0\cdot x_0+R\cdot g \qquad (3)$$

where elements in the materices are

$$M=\begin{bmatrix} m_{1,1} & & \\ & \ddots & 0 \\ & m_{i,i} & \\ 0 & & \ddots \\ & & m_{n,n} \end{bmatrix} \quad C=\begin{bmatrix} -\sum_{j=1}^{n+n_0}c_{j,1}, \cdots, c_{1,i}, \cdots, c_{1,n} \\ \ddots \\ c_{i,1}, \cdots, -\sum_{j=1}^{n+n_0}c_{j,i}, \cdots, c_{i,n} \\ \ddots \\ c_{n,1}, \cdots, c_{n,i}, \cdots, -\sum_{j=1}^{n+n_0}c_{j,n} \end{bmatrix}$$

$$C_0=\begin{bmatrix} c_{1,n+1}, \cdots, c_{1,n+n_0} \\ \vdots \quad\quad \vdots \\ c_{i,n+1}, \cdots, c_{i,n+n_0} \\ \vdots \quad\quad \vdots \\ c_{n,n+1}, \cdots, c_{n,n+n_0} \end{bmatrix} \quad R=\begin{bmatrix} r_{1,1}, \cdots, r_{1,ng} \\ \vdots \quad\quad \vdots \\ r_{i,1}, \cdots, r_{i,ng} \\ \vdots \quad\quad \vdots \\ r_{n,1}, \cdots, r_{n,ng} \end{bmatrix}$$

2. Time Integration

The state equation of the mathematical model was obtained as Eq. (2). The computer simulation is implemented by time integration. The formal solution is represented by convolutional integration (extended Duhamel's integration) using transition matrix ϕ, while the time functions inside the matrix, ϕ, are implicit, such that practical integration cannot be implemented. Therefore, using spectral resolution, this integration will be explicitly projected on the eigenspaces[2] as

$$x(t)=\sum_{i=1}^{n}P_i\cdot e^{\alpha i(t-t_0)}\cdot x(t_0)+\int_{t_0}^{t}\sum_{i=1}^{n}P_i\cdot e^{\alpha i(t-\tau)}\cdot f^*(\tau)d\tau \qquad (4)$$

where α_i is the eigenvalue of matrix $C^*=M^{-1}\cdot C$, P_i is corresponding projective operator matrix and f^* is the temperature input vector as in $f^*=M^{-1}\cdot f$.

When f^* is a random time function that cannot be defined analytically, converting it into a discrete time function and integrating the Δt time interval in linear or stair interpolation, two useful recurrence formula can be derived. The former is[2)]

$$x(k\Delta t)=\phi(\Delta t)\cdot x((k-1)\Delta t)+U_0\cdot f^*((k-1)\Delta t)+U_1\cdot f^*(k\Delta t) \quad (5)$$

where k is time number, $\phi(\Delta t)=\sum_{i=1}^{n}P_i\cdot e^{\alpha i\cdot\Delta t}$, $U_0=\sum_{i=1}^{n}P_i\cdot a_{i0}$ and $U_1=\sum_{i=1}^{n}P_i\cdot a_{i1}$ with the coefficients $a_{i0}=-(1/\Delta t)\cdot(1/\alpha_i)^2\cdot e^{\alpha i\cdot\Delta t}+(1/\Delta t)\cdot(1/\alpha_i)^2+(1/\alpha_i)\cdot e^{\alpha i\cdot\Delta t}$ and $a_{i1}=(1/\Delta t)\cdot(1/\alpha_i)^2\cdot e^{\alpha i\cdot\Delta t}-(1/\Delta t)\cdot(1/\alpha_i)^2-(1/\alpha_i)$. The latter is[2)].

$$x(k\Delta t)=\phi(\Delta t)\cdot x((k-1)\Delta t)+U_s\cdot f^*(k\Delta t) \quad (6)$$

where $U_s=\sum_{i=1}^{n}P_i\cdot(1/\alpha_i)\cdot(e^{\alpha i\cdot\Delta t}-1)$.

In addition, approximated integrations are also useful. One of them, the Crank-Nikolson scheme which is unconditionally stable and does not necessarily require all m_{ii} $(i=1, \cdots, n)$ to be non-zero, can be written as

$$x(k\Delta t)=\phi_C(\Delta t)\cdot x((k-1)\Delta t)+U_C\cdot f(k\Delta t) \quad (7)$$

where $\phi_C(\Delta t)=(M/\Delta t-C)^{-1}\cdot(M/\Delta t)$ and $U_C=(M/\Delta t-C)^{-1}$. Another, the explicit scheme used when large-sized matrix computation is difficult to implement, is written as

$$x(k\Delta t)=\phi_e(\Delta t)\cdot x((k-1)\Delta t)+U_e\cdot f(k\Delta t) \quad (8)$$

where $\phi_e(\Delta t)=E+\Delta t\cdot M^{-1}\cdot C$ and $U_e=\Delta t\cdot M^{-1}$. Note that this simple computation does not require the whole system concepts except in the analysis of transition stability[2)].

3. System Parameters

Capacity, m_{ii}, generalized conductance, c_{ij}, and free input ratio, r_{ij}, will be called system parameters. Although Galerkin's method is effective for the purpose of obtaining these parameters precisely and mathematically, less precise, but practical methods for these parameters will be introduced in this paper.

The space domain of real phenomenon is subdivided into "Control Volumes (CV_S)" for inspecting heat flow balances. Nodes are placed in these centers, and heat conductions among these nodes are regarded as steady state processes. Generalized conductance, c_{ij}, can therefore be calculated from the coefficients of heat transmission and basic theory. For capacity, m_{ii}, of the i-th node C.V., the heat capacitance or volume is assumed. For the purpose of the discussion, some examples will now be presented.

Figure 2 indicates a wall heat conduction system and the thermal network model. The third

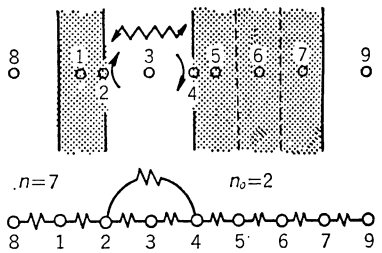

Fig. 2 Wall section with radiation between inner surfaces

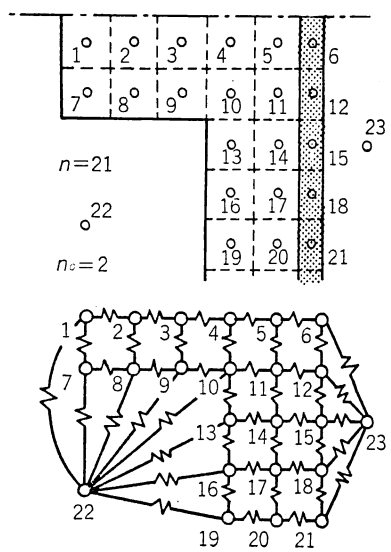

Fig. 3 Deformed wall section with multi-dimension heat flow

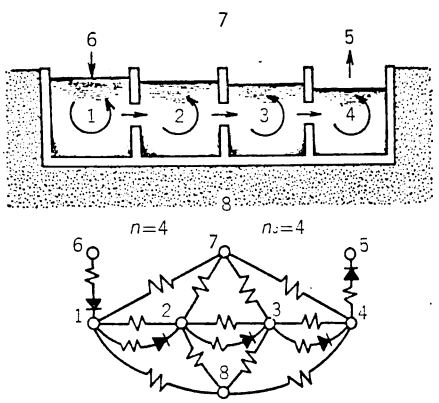

Fig. 4 Thermal accumulating vessels of cascade connection

node represents the air layer temperature, while the 2nd and 4th nodes represent the surface temperatures of both edges respectively. These capacitances are assumed to be $m_{22}=m_{44}=0$. Between these latter two nodes is a radiative heat transfer process which can be described by the Stephan-Boltzman law with form factors. Approximating linearity, radiative heat conduction, c_{ij}, can then be defined[4)].

Figure 3 illustrates a deformed wall section and the thermal network model. Thus, heat conductances among nodes can be calculated in a two-dimensional domain on the basis of the usual heat transmission evaluating method. This will also

hold true even in the three-dimensional domain.

Generalized conductances, c_{ij}, in the above example have the important characteristic of symmetry, namely $c_{ij}=c_{ji}$, which is unique to conduction systems. On the contrary, the temperature is transported by mass flow or advection in the example below.

Figure 4 represents the thermal accumulating vessels of a cascade connection and the thermal network model. The directed arc with the diode notation represents one-way generalized conductance by mass flow, which has the property of asymmetry, $c_{ij}=q$, with $c_{ji}=0$ defining the flow rate from nodes j to i as q (kcal/°C·hr).

The last example, as shown in Fig. 5, explains input ratio, r_{ij}, with regard to solar radiation. The figure symbolizes the six components of solar radiation, that is, g_1 (south), g_2 (north), g_3 (east), g_4 (west), g_5 (zenith) and g_6 (nadir) projected on a rectangular space coordinate, with the normal unit vector, n, on the receiving surface and the angles of azimuth, α, or tilt, β. Here, positive direction of angle, α is defined westward. The solar receiving surface is assumed to be the i-th node with an area of w and solar adsorptivities of a_1, a_2, a_3, a_4, a_5 and a_6 for each solar radiation component. The input ratios, r_{ij}, can then be calculated as follows:

a) if $\sin\beta\cdot\cos\alpha\geqq 0$, then
$r_{i,1}=a_1\cdot w\sin\beta\cdot\cos\alpha$ and $r_{i,2}=0$

b) if $\sin\beta\cdot\cos\alpha<0$, then
$r_{i,1}=0$ and $r_{i,2}=-a_2\cdot w\cdot\sin\beta\cdot\cos\alpha$

c) if $-\sin\beta\cdot\sin\alpha\geqq 0$, then
$r_{i,3}=-a_3\cdot w\cdot\sin\beta\cdot\sin\alpha$ and $r_{i,4}=0$

d) if $-\sin\beta\cdot\sin\alpha<0$, then
$r_{i,3}=0$ and $r_{i,4}=a_4\cdot w\cdot\sin\beta\cdot\sin\alpha$

e) if $\cos\beta\geqq 0$, then
$r_{i,5}=a_5\cdot w\cdot\cos\beta$ and $r_{i,6}=0$

f) if $\cos\beta<0$, then
$r_{i,5}=0$ and $r_{i,6}=-a_6\cdot w\cdot\cos\beta$

Consequently, the input ratios, r_{ij}, for the solar radiation components are constant in time if the building location and direction are fixed.

4. Network Mode Change

The change of generalized conductances, c_{ij}, or the change in number of n and n_0 without a change of the total number, $n+n_0$, will be defined as the network mode change. One of the physical meanings implied is that only the thermal connecting means or connecting intensities change without a vanishing or formation of material parts in the system. The other meaning, concerning space temperature, is that when fixed by the HVAC system, number n decreases and n_0 increases since the latter represents the total boundary number of the fixed nodes. In this way nonlinear problems can be solved in a narrow sense.

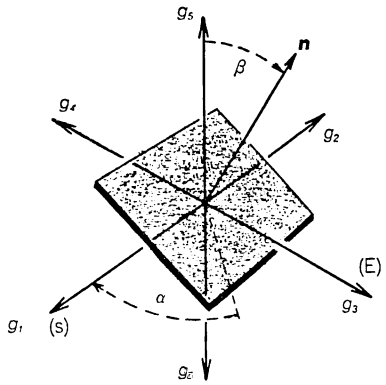

Fig. 5 Solar radiation components

Let us now consider the total kinds of network modes, m_d, appearing in the simulation period. These network modes can be numbered $m=1$ to m_d. In each mode, the transition matrix, ϕ, and drive matrix, U, should be computed in advance of simulation according to the theory described in Section 2. Therefore, simulation of network mode change can be implemented by time integration using transition and drive matrices according to each mode during the time interval, Δt.

The passive solar house in Fig. 6 is preferable to best explain these concepts practically. This building consists of an attached greenhouse, thermal accumulating rock bed, double skin and adobe wall. Solar control and insulation panels are represented by node 1, window panes by node 2, the adobe wall by nodes 10 to 12, the rock bed by nodes 17 to 20 and underground temperatures by nodes 21 to 23. Double skins are represented by nodes 4 to 6 and 7 to 9, with nodes 5 and 8 representing air layers in particular. Air in the rock bed is represented by nodes 13 to 16. Constant underground temperature and atmospheric air temperature are represented by nodes 25 and 26 respectively which are input variables. On the other hand, the space temperature of node 24 is variable between the input and state variables according to whether it is fixed by the HVAC system or not.

In this example various network modes can be realized by changing the air flow paths and the open or closed states of the solar control panel. Figure 7, for example, plots the heat accumulating mode circulating warmed air between the greenhouse and rock bed. Figure 8 illustrates the space heating mode releasing the stored heat in the rock bed.

The state vector, x, is then obtained primarily in each time integration step, and additional engineering index outputs, such as heating load or

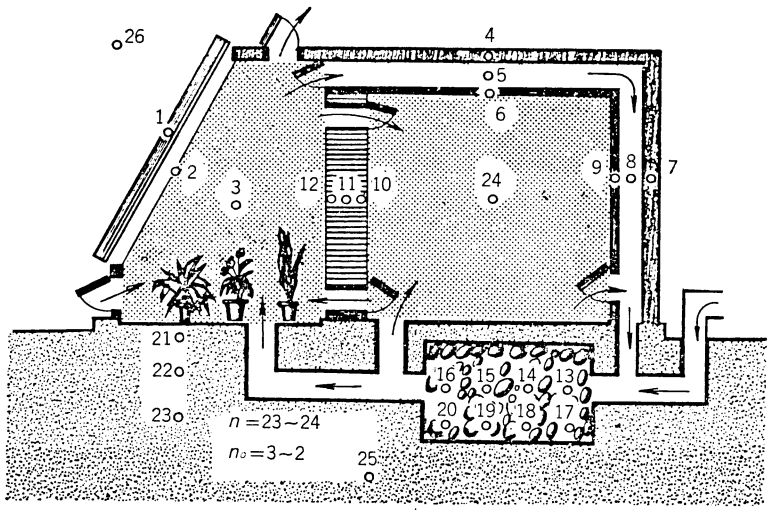

Fig. 6 Passive solar house

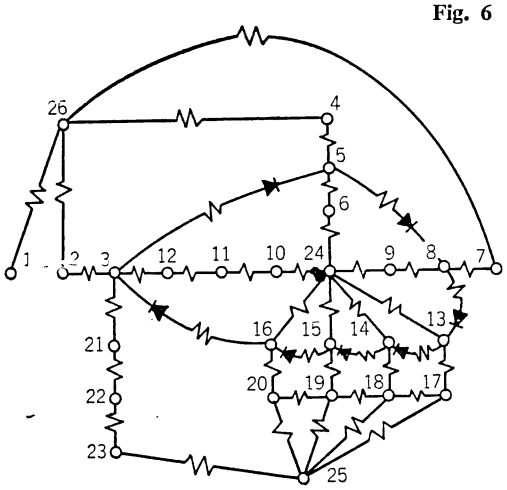

Fig. 7 Thermal accumulating mode

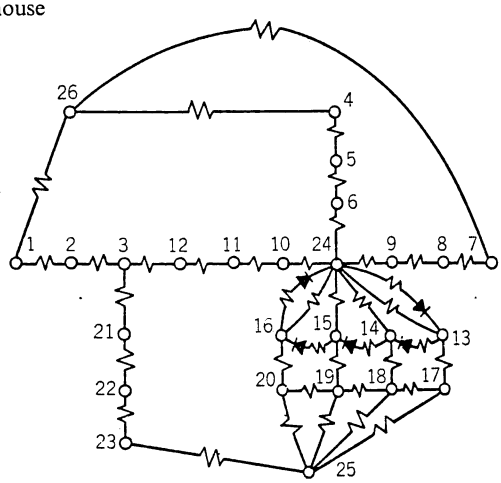

Fig. 8 Heating mode

environmental temperature, can be further computed by $y = F \cdot {}^t({}^t x, {}^t x_0, {}^t g)$, defining proper output vector, y, and matrix, F.

5. Condensation of State Equation

If the size of the state equation, n, is very large, the following approximately condensation method, which helps improve computation economy, can be used to reduce the size. Subdividing the state vector, x, into the master vector, x_m, and slave vector, x_s, accordingly subdividing the conductance matrix, C, Eq. (9) is written as ($v \neq 0$)

$$\begin{bmatrix} {}_{m,m}C & {}_{m,s}C \\ {}_{s,m}C & {}_{s,s}C \end{bmatrix} \cdot \begin{bmatrix} x_m \\ x_s \end{bmatrix} = \begin{bmatrix} v \\ 0 \end{bmatrix} \tag{9}$$

Since x_s is made a slave to x_m, Eq. (10) is obtained as

$$x_s = -{}_{s,s}C^{-1} \cdot {}_{s,m}C \cdot x_m = L \cdot x_m \tag{10}$$

Using Eq. (10) the approximated $\tilde{x}$ to x can be written as

$$\tilde{x} = \begin{bmatrix} E_m \\ L \end{bmatrix} \cdot x_m = V \cdot x_m \tag{11}$$

where E_m is the unit materix sized x_m. Weighted residual time integration along the period of $[0, T]$ is equated to 0, in which vector ${}^t\tilde{x}$ is also used for the weighting function, as in

$$\int_0^T {}^t\tilde{x} \cdot (M \cdot \dot{\tilde{x}} - C \cdot \tilde{x} - f) dt$$
$$= \int_0^T {}^t x_m \cdot ({}^t V \cdot M \cdot V \cdot \dot{x}_m - {}^t V \cdot C \cdot V \cdot x_m - {}^t V \cdot f) dt = 0 \tag{12}$$

This is identically satisfied by substituting

$${}^t V \cdot M \cdot V \cdot \dot{x}_m = {}^t V \cdot C \cdot V \cdot x_m + {}^t V \cdot f \tag{13}$$

which is the condensed state equation. In addition, decreasing the state equation size, by subdividing the system in place of this condensation will be proposed in the following section as the theory of coupling linear subsystems.

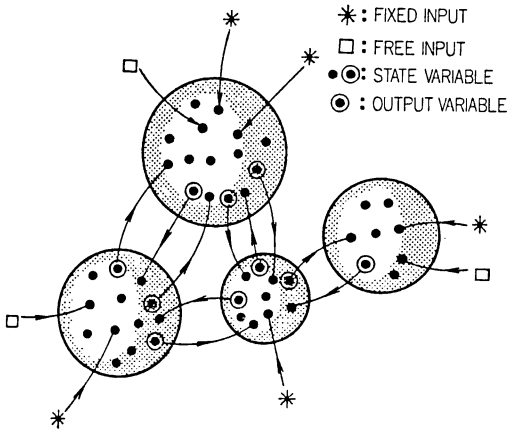

Fig. 9 Concepts of coupling subsystems

6. Theory of Coupling Subsystems

The state equation (14) can also be written with respect to the j-th subsystem, which is one of the total components constituting the entire system. If the component is a building or thermal storage tank, a detailed explanation has already been offered in Sections 3 and 4. In addition, even if the component exhibits non-linearity, linear approximation can be accomplished by defining the conductance matrix as a Jacobian matrix differentiated by the state vector. State equation (14) as described is then written as

$$\dot{x}_j(t) = C_j^* \cdot x_j(t) + f_j^*(t) \tag{14}$$

Recurrence formula (15), obtained after eigenvalue analysis of Eq. (14), is written as

$$x_j(k) = \phi_j(\Delta t) \cdot x_j(k-1) + U_{sj} \cdot f_j^*(k) \tag{15}$$

where the forcing vector, $f_j^*(t)$, is assumed to be a time step function for simplicity. The whole system state vector, X, can be defined by the direct sum of $X = x_1 \dot{+} x_2 \dot{+} \cdots\cdots \dot{+} x_{ns}$, where n_s is the total component number and x_j denotes the state vector of the j-th component. Vector x_j can be represented by vector X in Eq. (16) as

$$x_j(t) = D_{xj} \cdot X(t) \tag{16}$$

where matrix D will be called the relating matrix. The subspace spanned by the output variables in vector x_j will be defined as the output vector, y_j. Therefore, the output vector, Y, in the whole system will be defined by the direct sum of $Y = y_1 \dot{+} y_2 \dot{+} \cdots\cdots \dot{+} y_{ns}$. The forcing vector, f_j, is represented by the vector Y, fixed input x_0 and free input vector g in Eq. (17) as

$$f_j(t) = D_{yj} \cdot Y(t) + D_{0j}(t) \cdot x_0(t) + D_{gj} \cdot g(t) \tag{17}$$

Accordingly, the k-th time step state vector, $x_j(k)$, of the j-th component represented in Eq. (15) can be rewritten into Eq. (18) using the vectors X, Y, x_0 and g and the relationships of Eqs. (16) and (17) as

$$x_j(k) = \phi_j(\Delta t) \cdot D_{xj} \cdot X(k-1) + U_{uj} \cdot D_{yj} \cdot Y(k) + U_{uj} \cdot D_{0j} \cdot x_0(k) + U_{uj} \cdot D_{gj} \cdot g(k) \tag{18}$$

The output equation (19) can be derived from Eq. (18) as

$$y_j(k) = S_{xj} \cdot X(k-1) + S_{yj} \cdot Y(k) + S_{0j} \cdot x_0(k) + S_{gj} \cdot g(k) \tag{19}$$

by selecting out row equations from matrices in the right side, of which the row positions correspond to the points where y_j elements are placed in x_j. Here, matrix S will be called the submatrix. Vector $Y(k)$ is represented by either Eq. (20) or (20)′ as

$$Y(k) = y_1 \dot{+} \cdots \dot{+} y_{ns} = \begin{bmatrix} S_{x1} \\ \vdots \\ S_{xns} \end{bmatrix} \cdot X(k-1) + \begin{bmatrix} S_{y1} \\ \vdots \\ S_{yns} \end{bmatrix} \cdot Y(k) + \begin{bmatrix} S_{o1} \\ \vdots \\ S_{ons} \end{bmatrix} \cdot x_0(k) + \begin{bmatrix} S_{g1} \\ \vdots \\ S_{gns} \end{bmatrix} \cdot g(k) \tag{20}'$$

and

$$= \bar{S}_x \cdot X(k-1) + \bar{S}_y \cdot Y(k) + \bar{S}_0 \cdot x_0(k) + \bar{S}_g \cdot g(k) \tag{20}$$

using the direct sum of Eq. (19) from each component. Consequently $Y(k)$ can be solved as

$$Y(k) = W_x \cdot X(k-1) + W_0 \cdot x_0(k) + W_g \cdot g(k) \tag{21}$$

where matrices W are computed by Eqs. (23) to (25), using the output variable transition matrix, φ, computed by Eq. (22). These equations are listed as

$$\varphi = (E - \bar{S}_y)^{-1} \tag{22}$$

$$W_x = \varphi \cdot \bar{S}_x \tag{23}$$

$$W_0 = \varphi \cdot \bar{S}_0 \tag{24}$$

$$W_g = \varphi \cdot \bar{S}_g \tag{25}$$

Equation (21) is a recurrence formula for output variables in the whole system. Note that if the time integration interval, Δt, and operating mode are given, matrices W become constant. Therefore, when there are a finite number of operating modes, solving simultaneous equations at every time step is not required, relative to preparing these matrices before simulation. Whole system time integration can be implemented by repeating the following algorithm:

(i) Compute the whole system output vector, $Y(k)$, at the k-th time step by Eq. (21), using the whole system state vector $X(k-1)$ at the $(k-1)$-th time step and input vectors $x_0(k)$ and $g(k)$ at the k-th time step.

(ii) Compute the subsystem state vector, $x_j(k)$,

in each component using Eq. (18).

7. Summary and Conclusions

Thermal network concepts that can be used for automated and systematic method for casting the relevant problems into state-space form have been described. Time integration schemes of state equations were overviewed, and in particular, an exact analytical scheme using the eigenvalue analysis by the author was reintroduced.

Some of the features of heat transfer in buildings, such as multi-dimensionality or mass transfer, were indicated, and calculation methods for system parameters were respectively described. Furthermore, time varying and non-linearity characteristics of these thermal systems were also indicated, and for these problems, network mode change concepts were presented as being effective.

For computational efficiency, the system condensation method reducing the order of state equation, and for systematic modeling, the theory of coupling subsystems were deduced. This modeling and simulation method can be expected to become the basis for identification of system parameters[9] or optimum control theory in the future.

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