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# System Identification Theory of the Thermal Network Model and an Application for Multi-chamber Airflow Measurement

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*A comprehensive theory of multi-chamber air infiltration measurement using a single tracer gas is Introduced from the general stand point of system identification. The thermal network model can be applied not only to the temperature transfer and diffusion system but also to the tracer gas transfer system. This model is formulated mathematically in a state equation. The coefficients in the state equation represent airflow rates of infiltration. Two theories for estimating these coefficients are deduced from the least square. These are batch and successive identification. Further, the evaluation method of estimation errors that has been so far insufficient is described. Finally, the accuracy and the practicality of the theory are confirmed by the actual measurement system and experiments.*

## 1. INTRODUCTION

THE MEASUREMENT of building air infiltration is an important technique in evaluating the energy consumption of heating or the efficiency of contamination control. However the airflow rate of infiltration is difficult to measure directly, and therefore is usually estimated indirectly using a tracer gas. In this method, temporal variation of the gas concentration can be expressed by an ordinary differential equation by assuming the building interior to be a single zone and the tracer gas to mix uniformly in that zone. The rate of infiltration becomes a coefficient in this equation. By measuring the variation of the gas concentration, an indirect estimation of the air-flow rate of infiltration is obtained through this equation as a relating equation. This method is called a single-chamber air infiltration measurement method.

In the actual situation there are a number of rooms in a building, and even if the doors between rooms are kept open, the uniformity of the gas concentration is often not realized. Even if an attempt is made to carry out measurement of a single room, the gas concentrations in adjacent rooms are not necessarily equal to that of the open air. Under these actual conditions, measurement by a single-chamber method will result in a serious error. On the other hand, not only the airflow rate between the inside and outside of a building, but also the interzonal airflow rate is a valuable piece of information. Consequently the multi-chamber airflow measurement will be required.

In the case of single-chamber air infiltration measurement, there is only one airflow rate to be estimated and only one measurement of gas concentration. The number of unknown quantities coincides with the number of known quantities, therefore, the airflow rate can be obtained easily. However in the case of multi-chamber measurement, the number of airflows to be estimated can far exceed the number of rooms. The number of airflows becomes the number of rooms plus one squared, at the maximum. In short, a large number of airflow rates must be estimated from a relatively small number of gas concentration measurements. Here lies one of the great difficulties in the multi-chamber airflow measurement.

The multi-chamber airflow measurement methods currently under study may be broadly divided into two categories ; one using a single tracer gas, and the other using multiple tracer gases. By using multiple tracer gases, the number of equations can be increased to match the number of unknown airflows. Thus, the mathematical process to obtain the airflow rate becomes simple. However, in the case of multiple tracer gases, the measuring apparatus becomes complicated. Moreover, it cannot be applied to a building with a larger number of rooms than utilized gases. In the measurement method presented in this paper, a single tracer gas is used.

The conventional airflow measuring processes can also be classified by the method of tracer gas injection. They comprise a decay method, a constant-feed method and a constant-concentration method. The tracer gas injection may be regarded as excitation to the gas transfer and diffusion system, and the measurement of gas concentration as its response. Therefore, all three methods attach some conditions to the excitation and response. However, the measurement method of the present paper is outside of these three categories. The essential point of this method is that the adequate variation of gas concentrations can be secured either in time or in room-to-room relationship. To obtain this variation of gas

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concentration, the injection of tracer gases may be made at random.

Sinden [1] gave a fundamental theory on the transfer and diffusion of a single tracer gas in a multi-chamber building. The simultaneous ordinary differential equation of vector matrix form that he introduced describes the transition after stopping injection of tracer gas, but does not describe the variation during injection. In his method, a simultaneous equation with the airflow rate as an unknown quantity is obtained by taking as many measurements in the decay process of gas concentration as there are number of rooms plus one. It can therefore be regarded as a kind of decay method using a single tracer gas. However, variations between rooms are small in the decay process of tracer gas concentration. Therefore, some of the simultaneous equations may become linearly dependent and the equation unsolvable. Further, as this is not a statistical method, it is easily affected by measurement errors. Honma [2] also investigated the method of using a single tracer gas. He adopted the least square method, which he applied not to a set of simultaneous ordinary differential equations but to a single equation for every chamber. Therefore the coupling between chambers does not hold, and sometimes a correct solution cannot be obtained. Grimsrud *et al.* [3] performed experiments for the case of three chambers by using $N_20$ (nitrous oxide) as a tracer gas. The theory is the same as Sinden's. They stated that the results were not satisfactory. On the other hand, the method of using multiple gases has been tested by a number of researchers. However, never more than six kinds of gases seem to have been used.

The theory presented here is neither an improvement nor a development of these past results, but a new general and wide-ranging theory based on a viewpoint more comprehensive than these methods. In the theory of the present paper, the transfer and diffusion system of tracer gas concentration is expressed by a state equation in the system theory [4]. The airflow rates of infiltration are parameters in this state equation and the gas concentrations are the states. Thus, obtaining airflow rates is regarded as a type of system identification [5]. The theory of the present paper can be applied not merely to the tracer gas transfer system but also to the temperature transfer and diffusion system. Namely this theory is generally applicable to a heat transfer system as well. In the present paper, first of all, the mathematical model of the general transfer and diffusion system will be described. This model is a general nodal system suitable for computer utilization, which will be called the network model. Especially when applying this model to a temperature diffusion system, it will be called the thermal network model. By the definition of the parameters and their subscripts in the mathematical model, the network model is generally described as a state equation. Next, the estimation theory of the parameters in the state equation will be described. Based on the least square method, this leads to two kinds of identification methods, batch and successive. Further will be described the evaluation method of estimation errors that has been so far insufficient. Finally, the multi-chamber airflow measurement system that has been developed on the basis of this theory, and the verification experiment at the National Swedish Institute for Building Research and the experiment on an actual building conducted in Tokyo will be described.

## 2. STATE EQUATION OF THE NETWORK MODEL

It is important to define the mathematical model for the tracer gas transfer and diffusion before the development of an estimating theory for the airflow rates. For example, Axley [6] proposed a mathematical model of the contaminant dispersion system, constituting a set of simultaneous ordinary differential equations in a similar way to the finite element method. The matrices of the total system of equation are formed by adding up the submatrix for each element, resulting in a more complicated procedure than the present approach. Axlay states that his method enables a mixed idealization of microscopic and macroscopic models through the compatibility with the finite element method. However, the nodal equation for the unified system in the present approach can also realize the same goal in a much simpler manner.

The transfer system of tracer gas in a multi-chamber building is translated into a mathematical model using a nodal system. Each chamber has an air volume as a characteristic and is regarded as a node with gas concentration of state. The nodes are interconnected with airflow rates. Associated with each node, an ordinary differential equation is represented to describe the gas equilibrium equation. In this equation, coefficients are the chamber volumes, airflow rates and injected tracer gas concentration. The state variables are the chamber gas concentrations. The input variables are the flow rates of the injected tracer gas. For all chambers, a simultaneous ordinary differential equation holds. Several other forms of nodal equations have been also introduced including the work by Sinden [1]. However, most of these formulations lack in generality for the expression of boundary conditions.

The building heat transfer system is also translated into a mathematical model by using a nodal system. The actual heat transfer takes place in a continuous body, therefore, to be precise, the actual heat transfer system is a distributed parameter system. However, the approximation to a lumped parameter system through spatial discretization makes it easier to handle. As a means of discretization, there are a finite element method, a finite difference method and a control volume method. Models, by any of these methods, are considered a nodal system, and can be expressed by the same mathematical model. Therefore, compatibility can be realized between the models by using three kinds of discretization methods, and permitting interconnections as well [7]. The nodal system with such a generality is called the thermal network model. Having originated from the field of heat transfer, the conventional symbols of that field will be employed in the equation. For a chamber or a node, the following tracer gas or energy equilibrium equation holds:

$$m_{ii} \cdot \dot{x}_i = \sum_{j=1}^{n+n_0} c_{ij} \cdot x_j - \sum_{j=1}^{n+n_0} c_{ij} \cdot x_i + \sum_{j=1}^{ng} \cdot r_{ij} \cdot g_j, \quad (1)$$

where $m_{ii}$ denotes the volume of the $i^{th}$ chamber or the thermal capacity of the $i^{th}$ node, and $x_i$ denotes the gas concentration of the $i^{th}$ chamber or the temperature of the $i^{th}$ node, dot above $x_i$ denotes the time derivative. On the equation's right hand side, $c_{ij}$ represents the airflow rate from the $j^{th}$ chamber to $i^{th}$ chamber or the thermal conductance from the $j^{th}$ node to the $i^{th}$ node, $g_j$ represents the release flow rate from the $j^{th}$ gas injection device or the heat generation of the $j^{th}$ heater, and $r_{ij}$ represents the concentration of the gas released to the $i^{th}$ chamber from the $j^{th}$ gas injection device or the ratio of heat flow to the $i^{th}$ node from the $j^{th}$ heater. Lastly, $n$ is the number of chambers or the number of temperature dependent nodes. In the transfer system of tracer gases, $n_o$ signifies the number of outside air gas concentrations, which are independent and usually equal to one. In the case of the thermal system, $n_0$ signifies the number of nodes with independent temperature, for example, outside air temperature, etc. Further, $n_g$ denotes the number of gas injection devices or heaters. In general, $x_i$ will be called a state variable, and $g_j$ a free input variable. It should be noted that the sums of the right hand side of equation (1) are implemented for all nodes. Therefore the nodal equation of the present paper represents the unified system of all spatial dimensions.

There are three kinds of parameters $m_{ij}$, $c_{ij}$, $r_{ij}$. In the heat transfer system, the $c_{ij}$ concerning conduction, radiation, transmission, etc., has a feature of symmetry $c_{ij}=c_{ji}$, because, whichever temperature of the nodes $i$ and $j$ may be higher, the heat transfer produced is the same as long as the provided temperature difference is the same. This is based on a type of reciprocality principle. On the other hand, the $c_{ij}$ concerning a fluid flow has an asymmetric property. If heat is transported from the node $j$ to $i$ by a fluid flow (volume flow rate $q$, specific heat $c_p$ and density $\rho$), then $c_{ij} = c_p \cdot \rho \cdot q$, and $c_{ji} = 0$. Since the $c_{ij}$ represents several transfer forms in this unified manner, it will be called a generalized conductance. The parameter $r_{ij}$ will be called a free input coefficient, and the order of the subscripts $i$ and $j$ indicates the direction. Parameter $m_{ij}$ is generally defined also in the condition of $i \neq j$. In the finite difference model of a heat transfer system, control volume model, etc., $m_{ij}$ exists only in the condition of $i = j$, and the heat capacity of nodes will be $m_{ii}$. Further, in the tracer gas transfer system model of a multi-chamber case as well, the chamber volume is $m_{ii}$. However in the heat transfer finite element model, there exists a heat capacity $m_{ij}$ with $i \neq j$. This means that the heat flow into the node due to the temperature difference contributes to the temperature rise not only of that node itself, but also of the adjacent nodes. Such a parameter as $m_{ij}$ will be called a generalized capacity. Thus, equation (1) can be described in a more generalized form as follows:

$$\sum_{j=1}^{n} m_{ij} \cdot \dot{x}_j = \sum_{j=1}^{n+n_0} c_{ij} \cdot x_j - \sum_{j=1}^{n+n_0} c_{ij} \cdot x_i + \sum_{j=1}^{ng} r_{ij} \cdot g_j. \quad (2)$$

In equation (2), the first term on the right hand side shows the mass flow rate or the heat flow into the node, and the second term the flow out of the node. This representation is an unfamiliar one in the heat transfer field. By the use of the mass conservation law expressed by equation (3), equation (2) can be rewritten as equation (4):

$$\sum_{j=1}^{n+n_0} c_{ij} = \sum_{j=1}^{n+n_0} c_{ij}, \quad (3)$$

$$\sum_{j=1}^{n} m_{ij} \cdot \dot{x}_j = \sum_{j=1}^{n+n_0} c_{ij} \cdot (x_j - x_i) + \sum_{j=1}^{ng} r_{ij} \cdot g_j. \quad (4)$$

Equation (4) is the same as equation (2) as long as equation (3) holds true. The first term of the right hand side in equation (4) shows that the heat flow rate is proportional to the temperature difference in the case of a heat transfer system. The constraint of equation (3) is important in proving that the eigenvalues of the state equation for the general transfer and diffusion system have a negative real part. The state equation will be constituted by equation (4). By defining the $n$-dimensional state vector as $\mathbf{x} = {}^t(x_1, x_2, \ldots, x_n)$, $n_0$-dimensional assigned input as $\mathbf{x}_0 = {}^t(x_{n+1}, \ldots, x_{n+n_0})$ and $n_g$ - dimensional free input vector as $\mathbf{g} = {}^t(g_1, g_2, \ldots, g_{ng})$, and combining the nodal equations from node 1 to $n$, the following simultaneous ordinary differential equation holds. Where the superscript $t$ on the left shoulder of vector or matrix means the transpose according to the definition in traditional mathematical textbooks:

$$\mathbf{M} \cdot \dot{\mathbf{x}} = \mathbf{C} \cdot \mathbf{x} + \mathbf{C}_0 \cdot \mathbf{x}_0 + \mathbf{R} \cdot \mathbf{g} \quad (5)$$

where the $n \times n$ matrix $\mathbf{M}$ is called a capacity matrix, the $n \times n$ matrix $\mathbf{C}$ a conductance matrix, the $n \times n_0$ matrix $\mathbf{C}_0$ an assigned input matrix, and the $n \times n_g$ matrix $\mathbf{R}$ a free input matrix. It should be noted that each matrix element is expressed by notation defined in typical linear algebra. The row $i$, column $j$ element of the capacity matrix $\mathbf{M}$ is $m_{ij}$. Except diagonal elements, the row $i$ column $j$ element of the matrix $[\mathbf{C}, \mathbf{C}_0]$ is $c_{ij}$. The $i_{th}$ diagonal element is a summation of $-c_{ji}$ from $j = 1$ to $j = n + n_0$. The row $i$ column $j$ element of the free input matrix $\mathbf{R}$ is $r_{ij}$. These subscript definitions will lead us to compatibility with finite element modeling [7].

For the purpose of computer simulation of the tracer gas dispersion, first the parameters of airflow rates in equation (5) have to be defined deductively, and secondly the time integration of that equation has to be implemented. The former problem is solved by using another model of the airflow network [8]. For the latter problem of time integration the exact and analytical solution using spectral decomposition [9, 10] has been given by the author. The system identification described in the present paper is an opposite problem to these computer simulation methods and may be called the inverse problem in some cases.

## 3. MEASUREMENT EQUATION

The three kinds of coefficients $m_{ij}$, $c_{ij}$ and $r_{ij}$ will be called system parameters. Some of the system parameters in the state equation may be supposed to be known. A practical example of this in the tracer gas transfer system, is when each chamber is equipped with a mixing fan, the chamber volume $m_{ii}$, can be assumed to be known. Further, when the concentration of the injected tracer gas is measured, the $r_{ij}$ can be assumed to be known. An

example, in the case of the heat transfer system, is when the spatial discretization is made by the finite element method with the coefficient of the time derivative term in the partial differential equation of the unsteady state heat conduction problem taken as 1 , then the $m_{ij}$ thus obtained has a volumetric dimension. Accordingly, $m_{ij}$ will be known. Or, when excitation is given to the heat transfer system with an electric heater, the electric and thermal conversion factor is 1 , then $r_{ij}$ will be known. In this way, usually concerning a node, at least one parameter can be assumed to be known. The system parameters other than these are unknown and need to be estimated. In equation (5), by shifting the terms consisting of known parameters to the left hand side, we obtain the vector $\mathbf{y}(\dot{x}_i, x_i, g_i)$. The resultant matrices, when the known parameters are thus removed from $\mathbf{M}$,$\mathbf{C}$,$\mathbf{C}_0$,and $\mathbf{R}$ will be represented as $\tilde{\mathbf{M}}$,$\tilde{\mathbf{C}}$ ,$\tilde{\mathbf{C}}_0$, and $\tilde{\mathbf{R}}$ respectively:

$$\mathbf{y}=-\tilde{\mathbf{M}}\cdot\dot{\mathbf{x}}+\tilde{\mathbf{C}}\cdot\mathbf{x}+\tilde{\mathbf{C}}_0\cdot\mathbf{x}_0+\tilde{\mathbf{R}}\cdot\mathbf{g}. \tag{6}$$

Here will be defined the vectors of parameters that are to be identified. Assume that there are $n_m$ components of the parameters of $m_{ij}$ to be identified in the matrix $\mathbf{M}$, the vector, having these in an arbitrary element order, is defined as $\mathbf{m}(n_m)$. Similarly, the vector constituted of $c_{ij}$ to be identified in matrix $\mathbf{C}$ and $\mathbf{C}_0$, is $\mathbf{c}(n_c)$, and the vector constituted of $r_{ij}$ in matrix $\mathbf{R}$ is $\mathbf{r}(n_r)$. Thus, equation (6) is transformed into an explicit form of vectors $\mathbf{m}$, $\mathbf{c}$, $\mathbf{r}$:

$$\mathbf{y}=-\tilde{\mathbf{M}}\cdot\dot{\mathbf{x}}+[\tilde{\mathbf{C}},\tilde{\mathbf{C}}_0]\cdot\begin{bmatrix}\mathbf{x}\\ \mathbf{x}_0\end{bmatrix}+\tilde{\mathbf{R}}\cdot\mathbf{g}=\mathrm{D}(\dot{x}i)\cdot\mathbf{m}+\mathbf{X}(xi)\cdot\mathbf{c}+\mathbf{G}(g_i)\cdot\mathbf{r}. \tag{7}$$

Here the sizes of matrices $\mathbf{D}(\dot{x}_i)$, $\mathbf{X}(x_i)$, and $\mathbf{G}(g_i)$ are respectively $(n\times n_m)$, $(n\times n_c)$, and $(n\times n_r)$ The constituting algorithm of vector $\mathbf{y}$, matrices $\mathbf{D}$, $\mathbf{X}$, and $\mathbf{G}$ is described as follows.

$\mathbf{y}(\dot{x}_i, x_i, g_i)$ : let $m_{ij}$ be a known parameter, then add $m_{ij}\cdot\dot{x}_j$ to the $i^{\text{th}}$ element of $\mathbf{y}$. Let $c_{ij}$ be a known parameter, then, for $i$ or $j$ not larger than $n$, add $-c_{ij}\cdot x_j$ to the $i^{\text{th}}$ element of $\mathbf{y}$, and add $c_{ij}\cdot x_j$ to the $j^{\text{th}}$ element. Let $r_{ij}$ be a known parameter, then add to the $i^{\text{th}}$ element $-r_{ij}\cdot g_i$. Here, "adding" means summing up iteratively after first setting to $\mathbf{y}=0$.

$\mathbf{D}(\dot{x}_i)$: let the $k^{\text{th}}$ element in $\mathbf{m}$ be $m_{ij}$. Then, $-\dot{x}_j$ enters the row $i$, column $k$ in $\mathbf{D}$.

$\mathbf{X}(x_i)$ : let the $k^{\text{th}}$ element in $\mathbf{c}$ be $c_{ij}$ , then, for $i$ or $j$ not larger than $n$, $x_j$ enters row $i$, column $k$ in $\mathbf{X}$, and $-x_j$ enters row $j$, column $k$.

$\mathbf{G}(g_i)$ : let the $k^{\text{th}}$ element in $\mathbf{r}$ be $r_{ij}$ . Then, $g_i$ enters row $i$ column $k$ in $\mathbf{G}$.

Given the known parameters and the parameter vectors $\mathbf{m}$, $\mathbf{c}$, and $\mathbf{r}$ to be identified, then these $\mathbf{y}$, $\mathbf{D}$, $\mathbf{X}$, and $\mathbf{G}$ will be definitely determined. There exist constraints among the parameters to be identified. The most important of them is the mass conservation law of equation (3). By setting up this equation for $i=1, 2, \ldots, n$, assume this to be a system of homogeneous linear equations concerning the vector $\mathbf{c}$. At this time, $n$ elements of the vector $\mathbf{c}$ are represented by the remaining $n_c-n$ elements. The former is defined as $\mathbf{c}_s(n)$, and the latter as $\mathbf{c}_m(n_c-n)$. Consequently, $\mathbf{c}_s$ can be expressed by $\mathbf{c}_m$ as follows:

$$\mathbf{c}_s=\mathbf{L}_s\cdot\mathbf{c}_m. \tag{8}$$

The size of the matrix in this linearly dependent relation $\mathbf{L}_s$ is $(n\times(n_c-n))$. Matrix $\mathbf{L}_s$ is formed by utilizing a subroutine in the standard mathematical library, which calculates the relationship for expressing the non-basic column vector with basic column vector in regard to the general rectangular matrix. The elements of the matrix $\mathbf{L}_s$ are $-1$ , 0 or 1 . Next, c can be expressed with $\mathbf{c}_m$ by the following equation:

$$\mathbf{c}=\begin{bmatrix}\mathbf{L}_s\\ \mathbf{E}_s\end{bmatrix}\cdot\mathbf{c}^m=\mathbf{L}\cdot\mathbf{c}^m, \tag{9}$$

where $\mathbf{E}_s$ is a unit matrix of size $n_c-n$. Although this is the most important constraint in the tracer gas transfer system, non-negative constraint of the estimated parameters can also be considered. This is implemented by the non-negative least square method [11] supplemented with the linear programming. However, this constraint will be satisfied automatically in most cases as long as the measurement with good precision is carried out and the proper system identification model is used.

There are several more constraints in the heat transfer system. For example, in the case of the thermal conduction system, there is the symmetry constraint $c_{ij}=c_{ji}$, and from the symmetry of the volumetric matrix $\mathbf{M}$, arises the constraint $m_{ij}=m_{ji}$. By the use of such *a priori* information, it is desirable for the identification accuracy to reduce the size of the identifying parameter vector as much as possible. It is not only possible to reduce the number of identifying parameters, but also to resolve itself into the identification of several kinds of further basic parameters such as fundamental thermal properties. In the finite element method, the matrix $\mathbf{M}$ is obtained by adding up submatrices corresponding to each finite element, therefore, a parameter $m_{ij}$ can be expressed by a linear combination of several kinds of specific heat $c_p$ and specific density $\rho$. Assuming that the heat transfer system is composed of $m_1$ kinds of materials, by the $m_1$-dimensional vector $\lambda_{m1}={}^t(c_{p1}\cdot\rho_1, c_{p2}\cdot\rho_2, \ldots, c_{pm1}\cdot\rho_{m1})$, vector $\mathbf{m}$ can be expressed as follows:

$$\mathbf{m}=\mathbf{L}_{m1}\cdot\lambda_{m1}, \tag{10}$$

where $\mathbf{L}_{m1}$ is a constant matrix of size $(n_m\times m_1)$ and its $l^{\text{th}}$ column vector can be obtained by putting $c_{p1}\cdot\rho_1=1$, and the other $c_p\cdot\rho=0$, in the adding up the submatrix process of the finite element method. There are $m_1$ kinds of materials, so there are $m_1$ kinds of thermal conductivities $\lambda$. Assuming that there are $m_\alpha$ kinds of convective heat transfer coefficients, by the vector $\lambda_{m2}={}^t(\lambda_1, \lambda_2, \ldots, \lambda_{m1}, \alpha_1, \alpha_2, \ldots, \alpha_{m\alpha})$ of the size $m_2=m_1+m_\alpha$, vector $\mathbf{c}_m$ can be expressed in the same manner as follows:

$$\mathbf{c}_m=\mathbf{L}_{m2}\cdot\lambda_{m2}. \tag{11}$$

Furthermore, assuming that there are $m_3$ kinds of solar absorptivity, by defining $\lambda_{m3}={}^t(\alpha_1, \alpha_2, \ldots, \alpha_{m3})$, vector $\mathbf{r}$ is expressed by the following equation:

$$\mathbf{r}=\mathbf{L}_{m3}\cdot\lambda_{m3}. \tag{12}$$

By performing identification on the above $\lambda_{m1}$, $\lambda_{m2}$, or $\lambda_{m3}$, we can obtain thermal property values. Therefore, in the discussion hereafter we will consider only the con-

straints of the mass conservation law of equation (9). Even if the constraints from (10) to (12) are taken into account, that is merely to multiply their matrix $\mathbf{L}_m$ from the right side by **D**, **X** and **G** matrices. Equation (7) can be rewritten as follows:

$$\mathbf{y} = \mathbf{D}(\dot{x}_i)\cdot \mathrm{m} + \mathbf{X}(x_i)\cdot \mathbf{L}\cdot \mathbf{c}_m + \mathbf{G}(g_i)\cdot \mathbf{r}. \tag{13}$$

Further, for simplicity, the system parameters are lumped together, and expressed by the following equation (14), and the corresponding **D**, **X**·**L**, and **G** are expressed by equation (15):

$$^{t}\mathbf{a} = (^{t}\mathbf{m},\ ^{t}\mathbf{c}_m,\ ^{t}\mathbf{r}), \tag{14}$$

$$\mathbf{Z} = (\mathbf{D}, \mathbf{X}\cdot\mathbf{L}, \mathbf{G}), \tag{15}$$

where the size of **a** is $n_a = n_m + n_c - n + n_r$. Thus, equation (13) is finally simplified to the following equation:

$$\mathbf{y}(t) = \mathbf{Z}(t)\cdot \mathbf{a}. \tag{16}$$

This equation will be called a measurement equation with respect to **a**. **Z** will be defined as a measurement matrix, and **a** as a system parameter vector.

## 4. BATCH IDENTIFICATION

As the evaluating function for identification, the time integral of the equation error $\mathbf{e}(t)$ of the measurement equation (16) will be used. First, $\mathbf{e}(t)$ is expressed by the following equation:

$$\mathbf{e}(t) = \mathbf{y}(t) - \mathbf{Z}(t)\cdot \mathbf{a}. \tag{17}$$

Because it is most appropriate to use a scalar as an evaluating value, the evaluating function $J_s$ is defined as the time integral of quadratic form of $\mathbf{e}(t)$:

$$J_s(\mathbf{a}) = \int_0^T {}^{t}\mathbf{e}(t)\cdot \mathbf{W}(t)\cdot \mathbf{e}(t)\,\mathrm{d}t, \tag{18}$$

where $\mathbf{W}(t)$ will be called a weighting matrix ($n \times n$), and $T$ is the time used for identification. Then, **W** is used for an accurate identification, and how it is calculated will be described in the following section. The weighting matrix **W** is necessary for the Markov estimate that realizes such good properties as unbiasedness, consistency or effectiveness. Therefore, when an ordinary unit matrix is used for **W**, the identification result can be said only to be optimal value. Although the measurement data are inherently analog values, because the measurement device and data processing device are computerized, they are treated as time discrete digital values. For this reason, by first dividing the integral section of $[0, T]$ into $p$ parts, equation (19) is obtained:

$$J_s(\mathbf{a}) = \sum_{k=1}^{p} \int_{(k-1)\Delta t}^{k\Delta t} {}^{t}\mathbf{e}(t)\cdot \mathbf{W}(t)\cdot \mathbf{e}(t)\,\mathrm{d}t. \tag{19}$$

Next, this equation is approximated as follows:

$$J_s(\mathbf{a}) \equiv \sum_{k=1}^{p} \frac{1}{\Delta t^2} \left( \int_{(k-1)\Delta t}^{k\Delta t} {}^{t}\mathbf{e}(t)\,\mathrm{d}t \right) \cdot \left( \int_{(k-1)\Delta t}^{k\Delta t} \mathbf{W}(t)\,\mathrm{d}t \right) \cdot \left( \int_{(k-1)\Delta t}^{k\Delta t} \mathbf{e}(t)\,\mathrm{d}t \right). \tag{20}$$

The vector $\mathbf{e}(t)$ is expressed by $\mathbf{y}(t)$ and $\mathbf{Z}(t)$. The $\mathbf{y}(t)$ and $\mathbf{Z}(t)$ are composed $\dot{x}_i$, $x_i$ and $g_i$. The time integrals of these variables in the $[(k-1)\ \Delta t,\ k\Delta t]$ section are calculated by the following three equations.

$$_{b}x_{ik} = \int_{(k-1)\Delta t}^{k\Delta t} \dot{x}_i\,dt = x_i(k\Delta t) - x_i((k-1)\Delta t), \tag{21}$$

$$_{s}x_{ik} = \int_{(k-1)\Delta t}^{k\Delta t} x_i\,dt \equiv \frac{\Delta t}{2}\cdot (x_i(k\Delta t) + x_i((k-1)\Delta t)), \tag{22}$$

$$_{s}g_{ik} = \int_{(k-1)\Delta t}^{k\Delta t} g_i\,dt \equiv \frac{\Delta t}{2}\cdot (g_i(k\Delta t) + g_i((k-1)\Delta t)). \tag{23}$$

Equations (22) and (23) are of a table-form approximate integration. If $x_i$ and $g_i$ have a continuous variation, a more precise integration method can be adopted. With these integrated values $_{b}x_{ik}$, $_{s}x_{ik}$, and $_{s}g_{ik}$, the $\mathbf{Z}_k$ and $\mathbf{y}_k$, which are the integration of $\mathbf{Z}(t)$ and $\mathbf{y}(t)$ in the time section $[(k-1)\ \Delta t, k\Delta t]$, can be expressed as the following equation:

$$\mathbf{Z}_k = [\mathbf{D}(_{b}x_{ik}),\ \mathbf{X}(_{s}x_{ik})\cdot \mathbf{L},\ \mathbf{G}(_{s}g_{ik})] = [\mathbf{D}_k,\ \mathbf{X}_k\cdot \mathrm{L},\ \mathbf{G}_k], \tag{24}$$

$$\mathbf{y}_k = \mathbf{y}(_{b}x_{ik},\ _{s}x_{ik},\ _{s}g_{ik}). \tag{25}$$

Then, equations (17) and (18) can be approximated respectively by equations (26) and (27) as follows:

$$\mathbf{e}_k = \mathbf{y}_k - \mathbf{Z}_k\cdot \mathbf{a}, \tag{26}$$

$$J(a) = \sum_{k=1}^{p} \frac{1}{\Delta t^2} {}^{t}\mathbf{e}_k\cdot \mathbf{W}_k\cdot \mathbf{e}_k. \tag{27}$$

In order to obtain a, which minimizes the evaluating function $J(\mathbf{a})$, a least square method is applied. This is implemented by differentiating $J(\mathbf{a})$ with respect to **a** and setting to 0.

$$\frac{\partial J}{\partial \mathbf{a}} = \frac{\partial}{\partial \mathbf{a}} \sum_{k=1}^{p} \frac{1}{\Delta t^2} \cdot {}^{t}\mathbf{e}_k\cdot \mathbf{W}_k\cdot \mathbf{e}_k$$

$$= \frac{\partial}{\partial \mathbf{a}} \sum_{k=1}^{p} \frac{1}{\Delta t^2} \cdot {}^{t}(\mathbf{y}_k - \mathbf{Z}_k\cdot \mathbf{a})\cdot \mathbf{W}_k\cdot(\mathbf{y}_k - \mathbf{Z}_k\cdot \mathbf{a})$$

$$= \sum_{k=1}^{p} \frac{1}{\Delta t^2}\cdot(-2\cdot {}^{t}\mathbf{Z}_k\cdot \mathbf{W}_k\cdot \mathbf{y}_k + 2\cdot {}^{t}\mathbf{Z}_k\cdot \mathbf{W}_k\cdot \mathbf{Z}_k\cdot \mathbf{a}) = 0. \tag{28}$$

Thus the estimated value of **a**, i.e. $\hat{\mathbf{a}}$ can be computed as follows:

$$\hat{\mathbf{a}} = \left( \sum_{k=1}^{p} {}^{t}\mathbf{Z}_k\cdot \mathbf{W}_k\cdot \mathbf{Z}_k \right)^{-1} \cdot \left( \sum_{k=1}^{p} {}^{t}\mathbf{Z}_k\cdot \mathbf{W}_k\cdot \mathbf{y}_k \right). \tag{29}$$

## 5. WEIGHTING MATRIX W

**A** weighting matrix **W** is introduced to reduce the negative influence due to measurement errors. Here, let the measurement error variances of $x_i$, $g_i$, be $\sigma_{xi}^2$ $\sigma_{gi}^2$, respectively. The error variances of integrated values in equations (21)-(23), i.e. $_{b}\sigma_{xi}^2$ $_{s}\sigma_{xi}^2$ and $_{s}\sigma_{gi}^2$ can be cal-

culated by the error propagation law as follows:

$$ {}_b\sigma_{xi}^2 = 2 \cdot \sigma_{xi}^2 , \tag{30} $$

$$ {}_s\sigma_{xi}^2 \cong \frac{1}{2} \cdot \Delta t^2 \cdot \sigma_{xi}^2 , \tag{31} $$

$$ {}_s\sigma_{gi}^2 \cong \frac{1}{2} \cdot \Delta t^2 \cdot \sigma_{gi}^2 . \tag{32} $$

The vectors of the measurement values and their measurement error variance vectors are defined as follows:

$$ {}_b\mathbf{x}_k = {}^t({}_b x_{1k}, \ldots, {}_b x_{nk}), \tag{33} $$

$$ {}_b\sigma_x = {}^t({}_b\sigma_{x1}, \ldots, {}_b\sigma_{xn}), \tag{34} $$

$$ {}_s\mathbf{x}_k = {}^t({}_s x_{1k}, \ldots, {}_s x_{n+n_0 k}), \tag{35} $$

$$ {}_b\sigma_x = {}^t({}_b\sigma_{x1}, \ldots, {}_s\sigma_{x,n+n_0}), \tag{36} $$

$$ {}_s\mathbf{g}_k = {}^t({}_s g_{1k}, \ldots, {}_s g_{ng,k}) , \tag{37} $$

$$ {}_s\sigma_g = {}^t({}_s\sigma_{g1}, \ldots, {}_s\sigma_{g,ng}). \tag{38} $$

The values ${}_b\mathbf{x}_k$, ${}_s\mathbf{x}_k$, ${}_s\mathbf{g}_k$ are the results of the addition of the error vectors ${}_b\mathbf{s}_{xk}$, ${}_b\mathbf{s}_{xk}$, ${}_s\mathbf{s}_{gk}$, to the true value vectors respectively. Since the true value vector reduces the equation error of the state equation to 0, the equation error vector $\varepsilon_k$, caused by measurement error, can be expressed as the following:

$$ \begin{aligned} \varepsilon_k &= -\mathbf{M} \cdot {}_b\mathbf{x}_k + [\mathbf{C}, \mathbf{C}_0] \cdot {}_s\mathbf{x}_k + \mathbf{R} \cdot {}_s\mathbf{g}_k \\ &= -\mathbf{M} \cdot {}_b\mathbf{s}_{xk} + [\mathbf{C}, \mathbf{C}_0] \cdot {}_s\mathbf{s}_{xk} + \mathbf{R} \cdot {}_s\mathbf{s}_{gk} . \end{aligned} \tag{39} $$

If $\Lambda_0(n \times n)$ is used for the variance-covariance matrix of the equation errors caused by measurement errors, it is then the expectation matrix of $\varepsilon_k \cdot {}^t\varepsilon_k$:

$$ \begin{aligned} \Lambda_0 &= E(\varepsilon_k \cdot {}^t\varepsilon_k) = \mathbf{M} \cdot E({}_b\mathbf{s}_{xk} \cdot {}^t{}_b\mathbf{s}_{xk}) \cdot {}^t\mathbf{M} + [\mathbf{C}, \mathbf{C}_0] \\ &\quad \cdot E({}_s\mathbf{s}_{xk} \cdot {}^t{}_s\mathbf{s}_{xk}) \cdot {}^t[\mathbf{C}, \mathbf{C}_0] + \mathbf{R} \cdot E({}_s\mathbf{s}_{gk} \cdot {}^t{}_s\mathbf{s}_{gk}) \cdot {}^t\mathbf{R} \\ &= \mathbf{M} \cdot \mathrm{diag}({}_b\sigma_x \cdot {}^t{}_b\sigma_x) \cdot {}^t\mathbf{M} + [\mathbf{C}, \mathbf{C}_0] \\ &\quad \cdot \mathrm{diag}({}_b\sigma_x \cdot {}^t{}_s\sigma_x) \cdot {}^t[\mathbf{C}, \mathbf{C}_0] + \mathbf{R} \cdot \mathrm{diag}({}_s\sigma_g \cdot {}^t{}_s\sigma_g) \cdot {}^t\mathbf{R}. \end{aligned} \tag{40} $$

This employs the fact that the covariances among ${}_b\mathbf{s}_{xk}$, ${}_s\mathbf{s}_{xk}$, and ${}_s\mathbf{s}_{gk}$ are 0 and that the covariances among the elements within these vectors are also 0. The symbol diag (**X**) designates the matrix composed only of the diagonal elements of the matrix **X**.

In this way, the propagation law from the measurement error to the equation error is formulated by equation (40). According to the theory of Markov estimation, the weighted matrix **W** can be calculated by the following equation:

$$ \mathbf{W}_k = \Lambda_0^{-1} . \tag{41} $$

Here, an inconvenience is that the $\Lambda_0$ must be calculated by using the matrices **M**, **C**, $\mathbf{C}_0$ and **R** including the parameters to be estimated. In short, results are needed to obtain the results. To solve this problem, iterative convergent calculations are performed. First, set $\Lambda_0$ to the unit matrix. Next, by using the parameters obtained, calculate $\Lambda_0$ by means of equation (40), and perform the identification again. This process is to be repeated. However, under practical conditions, we might have to be satisfied to make an identification calculation with $\mathbf{W}_k$ as a unit matrix. In the successive identification of the next section, $\mathbf{W}_k$ is always set to a unit matrix.

## 6. SUCCESSIVE IDENTIFICATION

Let us deduce the temporal discrete system of the system parameter **a**. By replacing the time step number $p$ of equation (29) with $k$, it can be transformed into the following equation:

$$ \mathbf{a}_k = \left( \sum_{j=1}^{k} {}^t\mathbf{Z}_j \cdot \mathbf{W}_j \cdot \mathbf{Z}_j \right)^{-1} \cdot \left( \sum_{j=1}^{k} {}^t\mathbf{Z}_j \cdot \mathbf{W}_j \cdot \mathbf{y}_j \right), \tag{42} $$

with the symbols defined as follows:

$$ \mathbf{A}_k = \left( \sum_{j=1}^{k} {}^t\mathbf{Z}_j \cdot \mathbf{W}_j \cdot \mathbf{Z}_j \right)^{-1} , \tag{43} $$

$$ \mathbf{u}_k = \sum_{j=1}^{k} {}^t\mathbf{Z}_j \cdot \mathbf{W}_j \cdot \mathbf{y}_j . \tag{44} $$

Relative equations between $\mathbf{A}_{k-1}$ and $\mathbf{A}_k$, $\mathbf{u}_{k-1}$ and $\mathbf{u}_k$ can be described:

$$ \mathbf{A}_k^{-1} = \mathbf{A}_{k-1}^{-1} + {}^t\mathbf{Z}_k \cdot \mathbf{W}_k \cdot \mathbf{Z}_k \tag{45} $$

$$ \mathbf{u}_k = \mathbf{u}_{k-1} + {}^t\mathbf{Z}_k \cdot \mathbf{W}_k \cdot \mathbf{y}_k . \tag{46} $$

By applying the Woodbury's matrix inversion formula [12] to equation (45), the following equation is obtained:

$$ \begin{aligned} \mathbf{A}_k = \mathbf{A}_k - \mathbf{A}_{k-1} \cdot {}^t\mathbf{Z}_k \cdot (\mathbf{W}_k^{-1} + \mathbf{Z}_k \cdot \mathbf{A}_{k-1} \cdot {}^t\mathbf{Z}_k)^{-1} \\ \cdot \mathbf{Z}_k \cdot \mathbf{A}_{k-1} . \end{aligned} \tag{47} $$

Equation (42) is transformed by using equations (43)-(47):

$$ \begin{aligned} \mathbf{a}_k &= \mathbf{A}_k \cdot \mathbf{u}_k = (\mathbf{A}_{k-1} - \mathbf{A}_{k-1} \cdot {}^t\mathbf{Z}_k \\ &\quad \cdot (\mathbf{W}_k^{-1} + \mathbf{Z}_k \cdot \mathbf{A}_{k-1} \cdot {}^t\mathbf{Z}_k)^{-1} \cdot \mathbf{Z}_k \cdot \mathbf{A}_{k-1}) \cdot (\mathbf{u}_{k-1} + {}^t\mathbf{Z}_k \\ &\quad \cdot \mathbf{W}_k \cdot \mathbf{y}_k) = (\mathbf{E}_a - \mathbf{A}_{k-1} \cdot {}^t\mathbf{Z}_k \cdot (\mathbf{W}_k^{-1} + \mathbf{Z}_k \cdot \mathbf{A}_{k-1} \cdot {}^t\mathbf{Z}_k)^{-1} \\ &\quad \cdot \mathbf{Z}_k) \cdot \mathrm{a}_{k-1} + \mathbf{A}_k \cdot {}^t\mathbf{Z}_k \cdot \mathbf{W}_k \cdot \mathbf{y}_k . \end{aligned} \tag{48} $$

Further, the following symbols are defined:

$$ \Phi_k = \mathbf{E}_a - \mathbf{A}_{k-1} \cdot {}^t\mathbf{Z}_k \cdot (\mathbf{W}_k^{-1} + \mathbf{Z}_k \cdot \mathbf{A}_{k-1} \cdot {}^t\mathbf{Z}_k)^{-1} \cdot \mathbf{Z}_k , \tag{49} $$

$$ \mathbf{B}_k = \mathbf{A}_k \cdot {}^t\mathbf{Z}_k \cdot \mathbf{W}_k . \tag{50} $$

Then, equation (48) can be expressed as a recurrence formula as follows:

$$ \mathbf{a}_k = \Phi_k \cdot \mathrm{a}_{k-1} + \mathbf{B}_k \cdot \mathbf{y}_k . \tag{51} $$

Thus, the temporal discrete system concerning **a** is obtained through equations (49)-(51), where $\mathbf{E}_a$ denotes a $(n_a \times n_a)$ unit matrix. As described in the previous section, $\mathbf{W}_k$ is always set to a unit matrix of $(n \times n)$. The initial $\mathbf{A}_0$ takes the unit matrix $\mathbf{E}_a$ or its constant multiples. The identifiability is examined by the rank of $\mathbf{A}_k$ matrix. In this case, the structural identifiability must be considered separately from the measuremental identifiability. Unless a sufficient quantity of measurement data is available, identification cannot be completed, where this will be called a measuremental identifiability. On the other hand, however much measurement data is obtained, identification cannot be achieved if the known and given system parameters are not sufficient in number. This is called a structural identifiability. In airflow measurement, usually the volumes of each chamber are given. In general, the structural identifiability will be satisfied, if at least one system parameter is given for

every node. When executing a successive identification in real time, rank calculation of $\mathbf{A}_k^{-1}$ in parallel is not practical. However, in many cases, it is known indirectly by observing the estimate results of system parameters, when that rank has reached the full rank. Before reaching the full rank, the results will be too large, too small or oscillating between extreme values, but after gaining a sufficient number of measurement values, the estimate results will stabilize.

## 7. ERROR EVALUATION OF THE IDENTIFICATION

Most of the conventional researches into the multi-chamber airflow measurement were insufficient in the error evaluation of the estimate results. Similarly, the identification theory in the control engineering field is also insufficient in error evaluation. In this section, the evaluation method of the identification error will be described. By regarding the identification errors of system parameters as the propagation from the equation error, this propagation law is formulated. The estimated vector $\hat{\mathbf{a}}$ of system parameters is calculated by equation (29). Further, the error vector $\mathbf{e}_j$ of the equation is defined by equation (26). The expectation of this vector can be taken as $E(\mathbf{e}_j) = 0$ by the unbiasedness of the Markov estimate. Hence, the following equation holds:

$$\begin{aligned}
\hat{\mathbf{a}} - E(\hat{\mathbf{a}}) &= \left(\sum_{j=1}^{p} {}^{t}\mathbf{Z}_j \cdot \mathbf{W}_j \cdot \mathbf{Z}_j\right)^{-1} \cdot \left(\sum_{j=1}^{p} {}^{t}\mathbf{Z}_j \cdot \mathbf{W}_j \cdot \mathbf{y}_j\right) \\
&\quad - \left(\sum_{j=1}^{p} {}^{t}\mathbf{Z}_j \cdot \mathbf{W}_j \cdot \mathbf{Z}_j\right)^{-1} \cdot \left(\sum_{j=1}^{p} {}^{t}\mathbf{Z}_j \cdot \mathbf{W}_j \cdot \mathbf{E}(\mathbf{y}_j)\right) \\
&= \left(\sum_{j=1}^{p} {}^{t}\mathbf{Z}_j \cdot \mathbf{W}_j \cdot \mathbf{Z}_j\right)^{-1} \cdot \left(\sum_{j=1}^{p} {}^{t}\mathbf{Z}_j \cdot \mathbf{W}_j \cdot \mathbf{y}_j\right) \\
&\quad - \left(\sum_{j=1}^{p} {}^{t}\mathbf{Z}_j \cdot \mathbf{W}_j \cdot \mathbf{Z}_j\right)^{-1} \cdot \left(\sum_{j=1}^{p} {}^{t}\mathbf{Z}_j \cdot \mathbf{W}_j \cdot (\mathbf{Z}_j \cdot \mathbf{a})\right) \\
&= \left(\sum_{j=1}^{p} {}^{t}\mathbf{Z}_j \cdot \mathbf{W}_j \cdot \mathbf{Z}_j\right)^{-1} \cdot \left(\sum_{j=1}^{p} {}^{t}\mathbf{Z}_j \cdot \mathbf{W}_j \cdot (\mathbf{y}_j - \mathbf{Z}_j \cdot \mathbf{a})\right) \\
&= \left(\sum_{j=1}^{p} {}^{t}\mathbf{Z}_j \cdot \mathbf{W}_j \cdot \mathbf{Z}_j\right)^{-1} \cdot \left(\sum_{j=1}^{p} {}^{t}\mathbf{Z}j \cdot \mathbf{W}_j \cdot \mathbf{e}_j\right). \qquad (52)
\end{aligned}$$

Defining the error covariance matrix of the vector $\hat{\mathbf{a}}$ as $\mathbf{\Lambda}_a(n_a \times n_a)$, the following equation is obtained (using the property that $E(\mathbf{e}_i, {}^{t}\mathbf{e}_j) = [0]$ provided $i \neq j$):

$$\begin{aligned}
\mathbf{\Lambda}_a &= E\left[(\hat{\mathbf{a}} - E(\hat{\mathbf{a}})) \cdot {}^{t}(\hat{\mathbf{a}} - E(\hat{\mathbf{a}}))\right] = \left(\sum_{j=1}^{p} {}^{t}\mathbf{Z}_j \cdot \mathbf{W}_j \cdot \mathbf{Z}_j\right)^{-1} \\
&\quad \cdot \left(\sum_{j=1}^{p} {}^{t}\mathbf{Z}_j \cdot \mathbf{W}_j \cdot E(\mathbf{e}_j \cdot {}^{t}\mathbf{e}_j) \cdot \mathbf{W}_j \cdot \mathbf{Z}_j\right) \\
&\quad \cdot \left(\sum_{j=1}^{p} {}^{t}\mathbf{Z}_j \cdot \mathbf{W}_j \cdot \mathbf{Z}_j\right)^{-1}. \qquad (53)
\end{aligned}$$

Next, let us consider how to calculate $E(\mathbf{e}_j \cdot {}^{t}\mathbf{e}_j)$. In general, there are two causes of equation errors. One is the measurement error, and the other is the structural disagreement between the actual phenomenon and state equation model. If only the measurement error is taken into consideration, $E(\mathbf{e}_j \cdot {}^{t}\mathbf{e}_j)$ is equal to $\mathbf{\Lambda}_0$, expressed by equation (40). If, at this time, like equation (41), $\mathbf{W}_j$ is set to $\mathbf{\Lambda}_0^{-1}$, then it is evident that $\mathbf{\Lambda}_a$ is equal to $\mathbf{A}_k$ of equation (43).

Furthermore, the structural disagreement of the state equation model signifies the insufficient setting of the state equation size or structure, and this will appear concretely in the equation residual calculated by the following equation:

$$\mathbf{v}_j = \mathbf{y}_j - \mathbf{Z}_j \cdot \hat{\mathbf{a}}. \qquad (54)$$

If this structural disagreement is also concluded to be the identification error of system parameters, the identification error can be evaluated by the error propagation from the equation residual expressed by equation (54) to the identified parameter. This is obtained by setting $E(\mathbf{e}_j \cdot {}^{t}\mathbf{e}_j)$ to the expectation matrix of $\mathbf{v}_j \cdot {}^{t}\mathbf{v}_j$ quation (53). In order to make an unbiased estimate of this expectation matrix, by setting the degree of freedom equal to the number obtained by subtracting the parameter number $n_a$ from the total summing number $p$, the following equation can be used for calculation. This will be substituted into equation (53):

$$E(\mathbf{e}_j \cdot {}^{t}\mathbf{e}_j) \cong \frac{1}{p - n_a} \cdot \sum_{j=1}^{p} \mathbf{v}_j \cdot {}^{t}\mathbf{v}_j. \qquad (55)$$

The error variance of system parameters is positioned in the diagonal elements of $\mathbf{\Lambda}_a$, and the covariance in the non-diagonal elements.

Upon identification of the system parameters, the state equation is formed accordingly. Evaluation indices will be obtained which represent how well the actual phenomenon can be explained by this state equation. As a clue to this evaluation the sum of residual squares of the measurement equation is taken. Denoting this sum as $s(\mathbf{a})$, and by equations (54) and (42), the following equation can be obtained after a little manipulation:

$$\begin{aligned}
s(\hat{\mathbf{a}}) &= \sum_{j=1}^{p} {}^{t}\mathbf{v}_j \cdot \mathbf{W}_j \cdot \mathbf{v}_j = \sum_{j=1}^{p} {}^{t}\mathbf{y}_j \cdot \mathbf{W}_j \cdot \mathbf{y}_j \\
&\quad - \left(\sum_{j=1}^{p} {}^{t}\mathbf{y}_j \cdot \mathbf{W}_j \cdot \mathbf{Z}_j\right) \cdot \left(\sum_{j=1}^{p} {}^{t}\mathbf{Z}_j \cdot \mathbf{W}_j \cdot \mathbf{Z}_j\right)^{-1} \\
&\quad \cdot \left(\sum_{j=1}^{p} {}^{t}\mathbf{Z}_j \cdot \mathbf{W}_j \cdot \mathbf{y}_j\right). \qquad (56)
\end{aligned}$$

Two evaluation indices are available depending on what the ratio of $s(\hat{\mathbf{a}})$ is taken against. The first one is the index against the measurement error. Let this sum of residual squares be $s_0$, the following equation can be obtained in the same manner as equation (40):

$$\begin{aligned}
s_0 &= {}_{b}^{t}\sigma_x \cdot {}^{t}\mathbf{M} \cdot \left(\sum_{j=1}^{p} \mathbf{W}_j\right) \cdot \mathbf{M} \cdot {}_{b}\sigma_x + {}_{s}^{t}\sigma_x \cdot {}^{t}[\mathbf{C}, \mathbf{C}_0] \\
&\quad \cdot \left(\sum_{j=1}^{p} \mathbf{W}_j\right) \cdot [\mathbf{C}, \mathbf{C}_0] \cdot {}_{s}\sigma_x + {}_{s}^{t}\sigma_g \cdot {}^{t}\mathbf{R} \cdot \left(\sum_{j=1}^{p} \mathbf{W}_j\right) \cdot \mathbf{R} \cdot {}_{s}\sigma_g. \qquad (57)
\end{aligned}$$

Accordingly, the $f_0$ defined by the following equation

will be called the ratio of sum of residual square to measurement error:

$$f_0 = \frac{s(\hat{\mathbf{a}})}{s_0}. \tag{58}$$

The other evaluation index is the ratio of $s(\hat{\mathbf{a}})$ against the total variation of the measurement vector $\mathbf{y}_j$. This is based on the same idea as the so-called coefficient of determinant. First, the weighted average of $\mathbf{y}_j$ is defined as $\bar{\mathbf{y}}$ and calculated by the following equation:

$$\bar{\mathbf{y}} = \left( \sum_{j=1}^{p} \mathbf{W}_j \right)^{-1} \cdot \left( \sum_{j=1}^{p} \mathbf{W}_j \cdot \mathbf{y}_j \right). \tag{59}$$

Then, the total variation $s_y$ can be calculated by the following equation:

$$s_y = \sum_{j=1}^{p} {}^{t}(\mathbf{y}_j - \bar{\mathbf{y}}) \cdot \mathbf{W}_j \cdot (\mathbf{y}_j - \bar{\mathbf{y}})$$

$$= \sum_{j=1}^{p} {}^{t}\mathbf{y}_j \cdot \mathbf{W}_j \cdot \mathbf{y}_j - \bar{\mathbf{y}} \cdot \left( \sum_{j=1}^{p} \cdot \mathbf{W}_j \right) \cdot \bar{\mathbf{y}}. \tag{60}$$

Hence, the coefficient of determinant $f_v$ can be calculated by equation (61):

$$f_v = 1 - \frac{s(\hat{\mathbf{a}})}{s_y}, \tag{61}$$

where $f_v$ is not defined when there is a 0 element constantly within $\mathbf{y}$.

It is possible to perform a testing hypotheses of the state equation model using these indices. A chi-square test is used based on $f_0$ to determine whether the residual is adequate or not by considering the measurement error. F-testing can be performed based on $f_v$ to determine whether the state equation model is significant or not when viewed from the equation residual alone. Adopting $f_0$ as the evaluation index is not practical due to the fact that $s_0$ is dependent upon the estimated system parameters, and also, it is unrealistic to assume that the cause of the equation residual is only the measurement error. An important premise is the linearity or invariability of the state equation model. In the case of air infiltration measurement, although the airflow rate should not vary greatly during the identification period, variation to some extent must be admitted. Further, the uniformity of gas concentration is assumed within a chamber, however, sometimes this assumption does not hold unless a mixing fan is used. These actual situations produce the equation residual. Consequently, if the $f_v$ value is not good, it is necessary to examine whether or not these premises and assumptions hold acceptably.

## 8. EXECUTION PROCEDURE OF SYSTEM IDENTIFICATION

In most cases, the theory of this system identification is executed based on measurement data. Further, successive identification becomes useful by its execution in real time. Therefore, it is advantageous to use a microcomputer in the operation process with flexible disks as memory devices.

First, the batch identification will be described. The measurement values are obtained time discretely. Therefore, $\mathbf{x}_k$, $\mathbf{x}_{ok}$, and $\mathbf{g}_k$ are time series with subscripts $k = 1, 2, \ldots, p$ representing time steps. After a sufficient lapse of measurement time, the batch identification is performed as follows.

First, set $\hat{\mathbf{a}}_0$ to 0.

(i) In case of the first identification, the weighting matrix $\mathbf{W}_k$ is set to the unit matrix $\mathbf{E}_n$. For subsequent identifications, $\mathbf{\Lambda}_0$ and $\mathbf{W}_k$ are determined by the identification result $\hat{\mathbf{a}}_0$ obtained in the previous step and equations (40) and (41), then the estimation value of $\hat{\mathbf{a}}_1$ at the current time step is calculated by equation (29). (ii) If $\| \hat{\mathbf{a}}_1 - \hat{\mathbf{a}}_0 \| <$ (allowable error), then the identification is taken to be complete. Otherwise, set $\hat{\mathbf{a}}_0 \Leftarrow \hat{\mathbf{a}}_1$, and repeat the calculation by returning to (i).

However, when the characteristics of the measurement error are not well known, or when the structure itself of the state equation is considered inadequate, it will be acceptable even if the optimal solution is obtained with the weighted matrix as a unit matrix.

Next, successive identification will be described. This is to proceed with identification iteratively every time the measurement is made at a certain time interval $\Delta t$. In this case, the discrete time system involving $\mathbf{a}_k$ in equation (51) is used. Where, the weighted matrix $\mathbf{W}_k$ is always set to the unit matrix $\mathbf{E}_n$. Further, the initial $\mathbf{A}_0$ is set either to the unit matrix $\mathbf{E}_a$ or the same multiplied by a proper coefficient. By giving the initial value $\mathbf{a}_0$, the following computing procedure is repeated from $k = 1$ : (i) obtain the $k^{\text{th}}$ time step measurement value, and calculate $\mathbf{A}_k$ from equation (47) ; (ii) calculate $\Phi_k$, $\mathbf{B}_k$ by equations (49) and (50) ; (iii) calculate $\mathbf{a}_k$ from $\mathbf{a}_{k-1}$ by equation (51); (iv) increase $k$ by 1 , return to (i) and repeat.

Since only the data from the immediately preceding time step is required to complete a calculation, the entire past measurement data need not be retained. In this calculation, the matrix inversion of the size of the number of nodes n is required. Therefore, the quick calculation performance is important when using a microcomputer. For the purpose of following the time variations of the system parameters to be identified, it is necessary to devise some function of forgetting the past. One of the alternatives is to replace all $\mathbf{A}_{k-1}$ by $\mathbf{A}_{k-1}/w^2$ in the right hand side of equation (47), where coefficient w is defined in $1 \geqslant w > 0$.

## 9. MULTI-CHAMBER AIRFLOW MEASUREMENT SYSTEM

This system is composed of a device which creates a temporal variation of tracer gas concentrations for each chamber, a device to measure the injection flow rate of the gas and the gas concentrations of each chamber continuously, and a device to analyse measurement data in real time. Figure 1 shows the configuration of this system. The mass flow controller is a device which controls the injection flow rate of the tracer gas and simultaneously measures it. Three microcomputers, CPU-1 , CPU-2, and CPU-3 are employed, which have the following functions respectively. Fitst, the CPU-1 operates:

(a) to open/close the solenoid valves by selecting chamber numbers for tracer gas injection according to schedule or by random numbers;

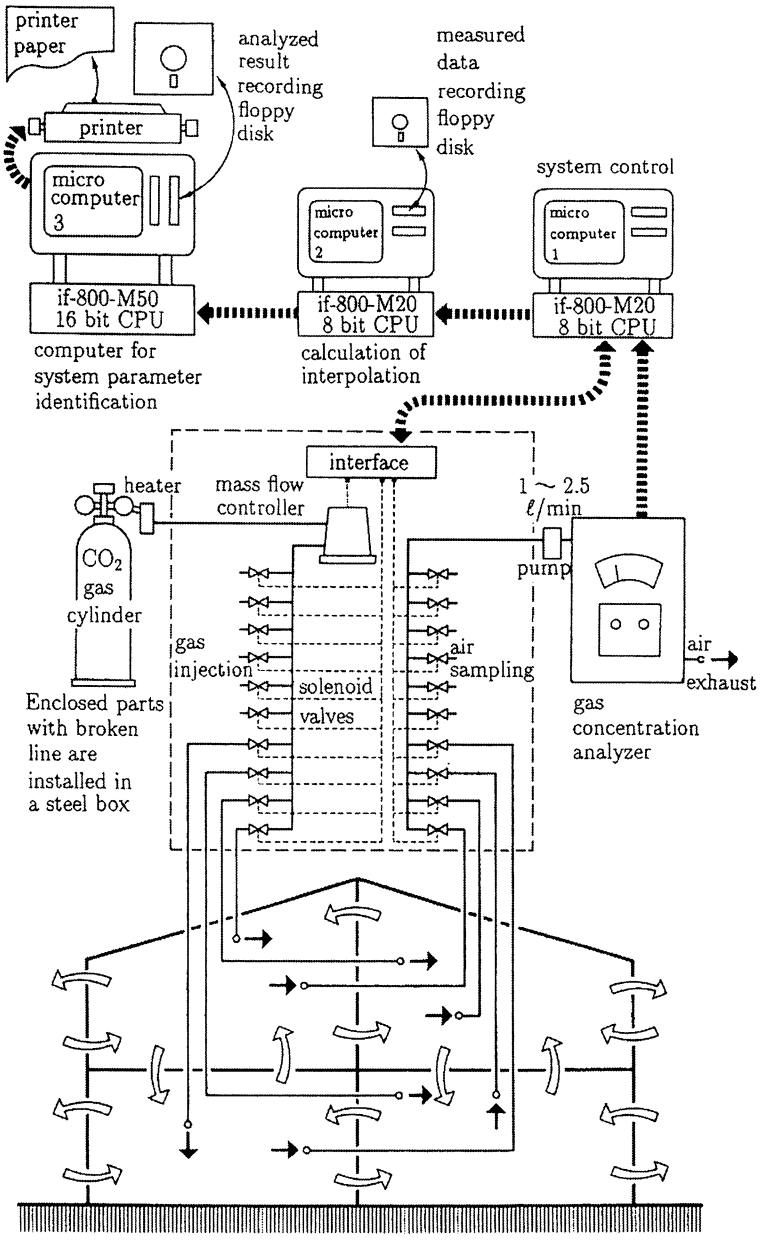


Fig. 1. Diagram of multi-chamber airflow measurement system

(b) to control the mass flow controller by determining the tracer gas injection flow rate according to schedule or by random numbers;
(c) to open/close the solenoid valves by selecting the chamber numbers for gas sampling according to schedule;
(d) to collect the data of gas concentration, the chamber number, injection flow rate and the chamber number, to relay these data to the CPU-2.

It takes approximately one minute for the analyser to obtain a concentration measurement of a chamber, due to the response lag and the purging time ofthe air remaining inside the tube. Therefore, the CPU-2 operates:

(e) to achieve the simultaneous concentration data for all chambers by performing an interpolation calculation for temporal forward and backward;
(f) while recording the data of these concentrations and

injection flow rates, etc. successively on the floppy disk, and relaying these data to the CPU-3. And the CPU-3 operates:

(g) to execute the successive identification in real time for the measurement data being received from the CPU-2 at time intervals $\Delta t$, and recording the analysis results on flexible disks, and also outputting them to the printer at proper time intervals.

This measurement system evolved through experimentation in 1984. Some experiments were carried out on existing buildings to investigate the applicability of the system identification theory. The verification of the theory itself was performed by a numerical experiment to estimate system parameters such as heat conductances on the finite element model of the heat transfer system [13].

The present measurement system has several bulky components such as, a box containing solenoid valves and mass flow controller, a gas concentration analyser, and three desktop computers. A practical problem in future is to develop a compact measurement system suitable for field measurement.

## 10. VERIFICATION EXPERIMENT

After several measurements in actual buildings, the measurement system had not been used. In 1987, the author had an opportunity of visiting Sweden by the invitation of Prof. Tor-Göran Malmström of the Stockholm Royal Institute of Technology. The author had already learned from reading *Building and Environment* that Dr Mats Sandberg, who works in the National Swedish Institute for Building Research, has access to a ventilation test house. Upon proposing the verification experiment of this measurement system to Dr Sandberg, the author received his willing consent. While staying in Sweden from November 1987 to May 1988, the author was engaged in this experiment, spending most of the period at the National Institute in Gävle. Although the theory itself is correct, it was interesting to study the effect upon the estimated result of the measurement error of the gas concentration analyser and mass flow controller, and the error due to the interpolation calculation approximation of the gas concentration.

Figure 2 shows the global view of the test house. This test house is used for the study of airflow within a room or between rooms, etc. [14]. The author conducted the experiment by slightly modifying the test environment. Figure 3 shows a plan of the test house. Two of the rooms were used for the simplest case. Six airfiows in this model were simulated by using six air ducts. Because the installation of fans on all air ducts requires sophisticated room pressure control system, only the two ducts exposed to the outside air were large diameter ducts without fans. By this means, the room pressure becomes slightly higher than that of the laboratory hall, thus preventing infiltration airflow from the outside of the test house which cannot be measured. Airflow rates inside the ducts were measured, and by taking this as a reference value, error evaluation of this measurement system was performed. However, as it is difficult to measure directly the airflow

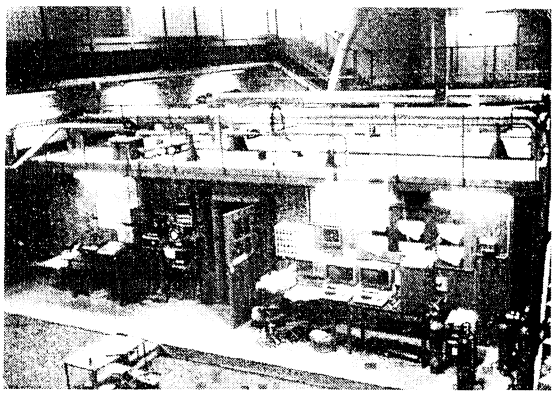

Fig. 2. View of the test house.

rate of the two large-diameter ducts, these were calculated from the other four airflow rates by the law of continuity. The measurement of airflow rates was made by using the bend duct pressure difference [15] . Each room was provided with two mixing fans to make the gas concentration uniform within the room.

The gas injection cycle was set to 1 h. The measured results are shown in Figs 4 and 5. The gas concentration variations due to the gas injection in each room are shown in Figs 6 and 7. The measured concentration of the outdoor air was constant at about 300 ppm. The estimated system parameters are the effective mixing volumes $m_{ii}$, ($dm^3$) and airflow rates $c_{ij}$ ($dm^3\ s^{-1}$). Figure 8 shows the results of batch identification. Directly measured values are in parentheses ( ), and will be the evaluation reference for estimation errors. The estimated inter-room airflow rates are smaller than the actual values. The largest error is about 20% against the true value. Table 1 shows the error estimation results obtained from equation (53). The size relationship of the error variance is not estimated sufficiently. Figures 9 and 1 0 are the results of successive identification, from which it is recognized that the quantity of measurement data necessary for the identification was obtained in about 40 min. Further, after stabilization in the results of successive identification, about four small changes are observed. These changes agree with the points where the linear approxi-

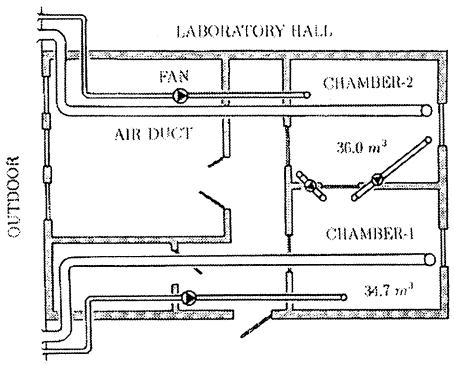


Fig. 3.Plan of the test house.

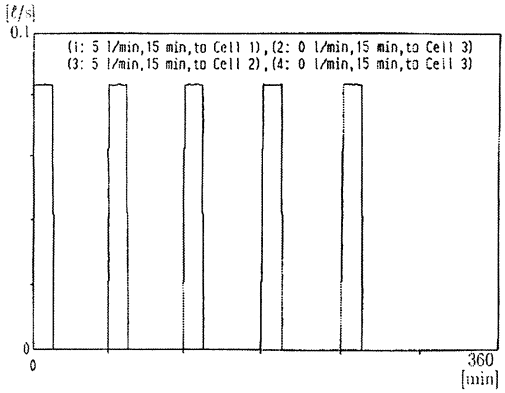


Fig.4. Tracer gas injection flow rate to chamber- 1.

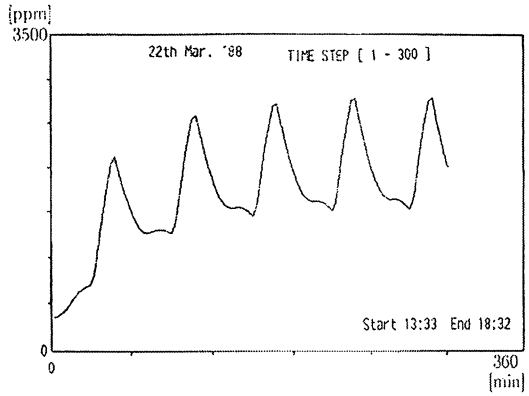


Fig.7.Tracer gas concentration in chamber-2.

mation error becomes maximum, which are those points obtained by *linear interpolation* of the three minutes interval concentration measurements. Therefore, this approximation may be considered to have a great and adverse influence on the estimation. Besides, in a separate experiment where mixing fans were not used, an estimation error as large as 90% against the true value occurred.

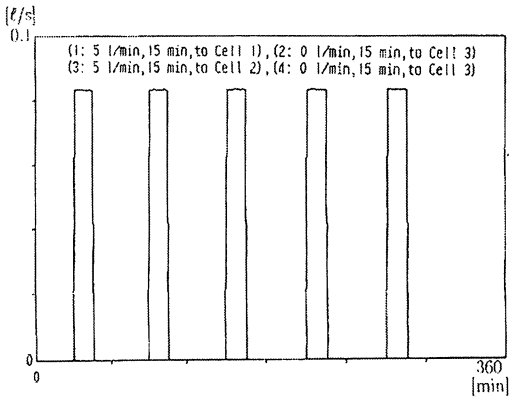


Fig.5. Tracer gas injection flow rate to chamber-2.

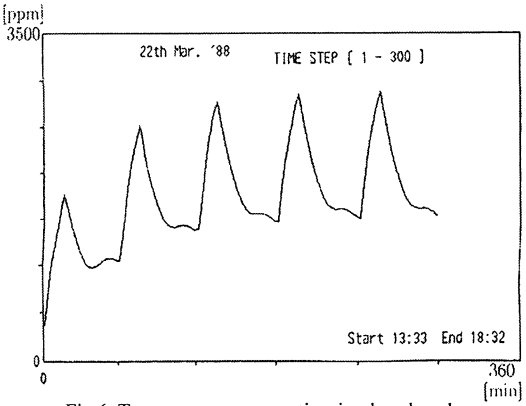


Fig.6. Tracer gas concentration in chamber-1.

## 11. FIELD EXPERIMENT IN ACTUAL BUILDING

In 1984, an experiment was conducted on a two-storied, old, wood house located at Okusawa, Setagayaku, Tokyo. Figure 11 shows the plan of the building. The room numbers, as defined in the identification model, are circled. For example, room No. 8 is the stairway leading to the second floor, and the room No. 10 represents the outside air. The measurement device was installed in the kitchen room, and its door

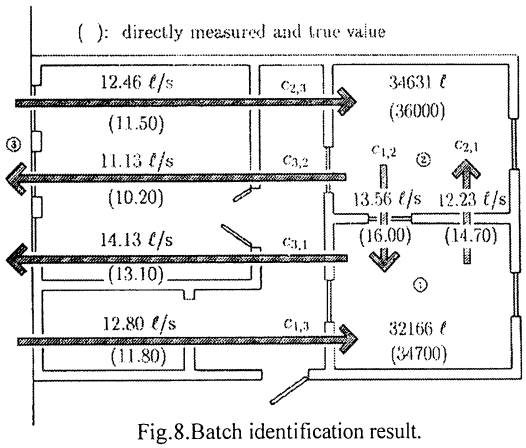


Fig.8.Batch identification result.

Table1.Error estimation

| | Estimated error in standard deviation |
|---|---|
| $\sigma m_{1,1}$ | 1371.8 $dm^{-3}$ |
| $\sigma m_{2,2}$ | 2432.1 $dm^{-3}$ |
| $\sigma c_{3,2}$ | 3.464 $dm^3 s^{-1}$ |
| $\sigma c_{3,1}$ | 3.489 $dm^3 s^{-1}$ |
| $\sigma c_{2,3}$ | 1.494 $dm^3 s^{-1}$ |
| $\sigma c_{2,1}$ | 2.939 $dm^3 s^{-1}$ |
| $\sigma c_{1,3}$ | 0.905 $dm^3 s^{-1}$ |
| $\sigma c_{1,2}$ | 1.773 $dm^3 s^{-1}$ |

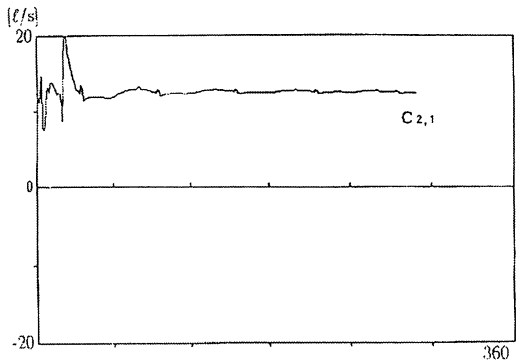


Fig.9.Successive identification airflow rate from chamber-i to chamber-2.

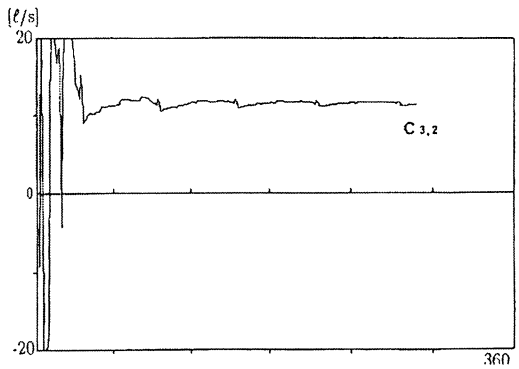


Fig. 10. Successive identification airflow rate from chamber-2 to outdoor.

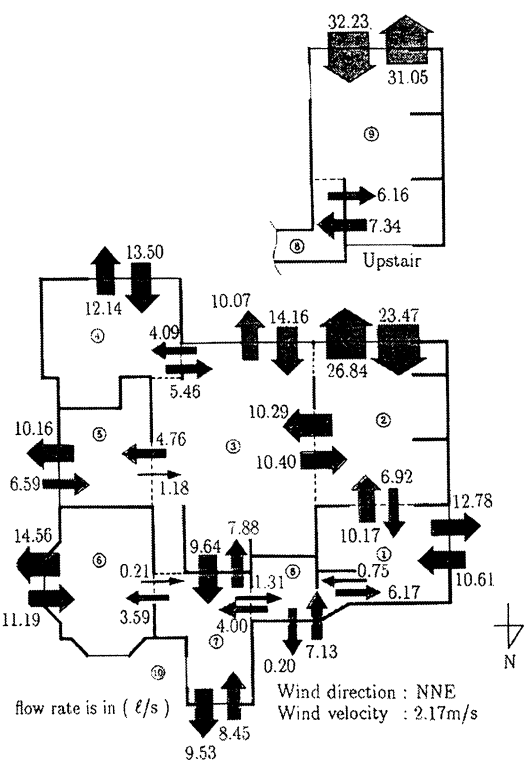


Fig.11. Batch identification result.

connected to the room No. 1 was sealed. The kitchen is excluded from this drawing. During the measurement, all the windows and doors facing the open air and the sliding doors between rooms were closed but not sealed. This building was air loose, and had much infiltration airflow. The air in each room was equally sampled from six different spots, and after mixing with a branching tube connector the air was led to the gas concentration analyser. The air inside the room was mixed with a fan of around 10 cm in diameter, attached at the tip of the tracer gas injection tube. The arrangement was thus, so that the assumption of uniformity in concentration is realized in each room. The injection of the gas was carried out according to a previously decided schedule. The tracer gas used was carbon dioxide. The analyser has a range of measurement. An injection schedule which does not exceed this range was determined with the rough guidance of air change rate generally obtained through experiences.

The parameters given for identification are the volume of each room $m_{ii}$ and the gas concentration $r_{ij}$. Therefore, $g_i$ represents the injection flow rate. The time interval $\Delta t$ of data sampling is one minute, and the successive identification was also performed with a one minute time step. Figures 12 and 13 show the temporal variations of the tracer gas injection flow rates into rooms 3 and 8 respectively. The variations of gas concentrations in these

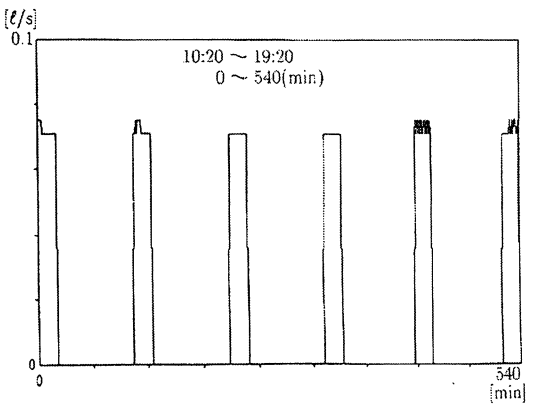


Fig.12. Tracer gas injection flow rate to chamber-3.

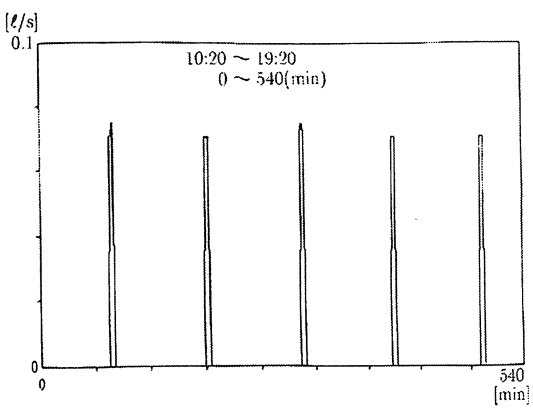


Fig.13. Tracer gas injection flow rate to chamber-8.

rooms are shown in Figs 14 and 15. Since there are 10 rooms in total, including the open air, the gas concentration was measured at 9 mm intervals for each room, and points within these intervals were interpolated. Consequently, because of the linear change in the interpolated points, a perfect agreement with the actual variation can not be assured.

Figures 16 and 17 show respectively the results of successive identifications of airflows from room 6 to the outside air and from the outside air to room 6. For around the first hour, sometimes the results exceed the high and low limits of the graph, however, they gradually come to a settlement. In other words, around this amount of time is required for identification. The results of the batch identification are shown in Fig. 11. These are the processed results based on all measurements of the initial 260 time steps. Further, in this batch identification, the coefficient of determinant calculated was 0.260. The estimated error standard deviation of airflow rates obtained by the residual analysis of the measurement equation is expressed in Table 4. Another cause of this error, besides the measurement and interpolation error, is the temporal change of airflows.

Some means to improve the performance of the measurement system can be proposed. The error produced by the linear interpolation is reduced by shortening the time interval $\Delta t$. Therefore the use of a quick response gas analyser will contribute to the improvement of pre-

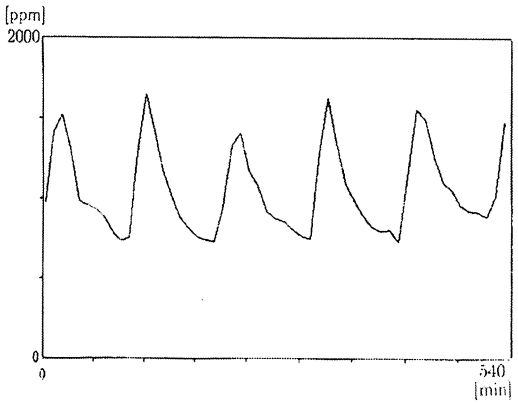


Fig.14. Tracer gas concentration in chamber-3.

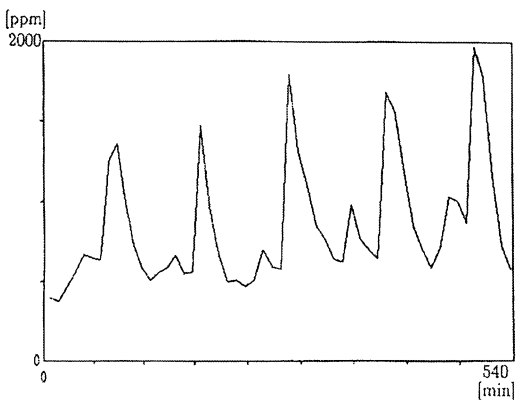


Fig.15. Tracer gas concentration in chamber-8.

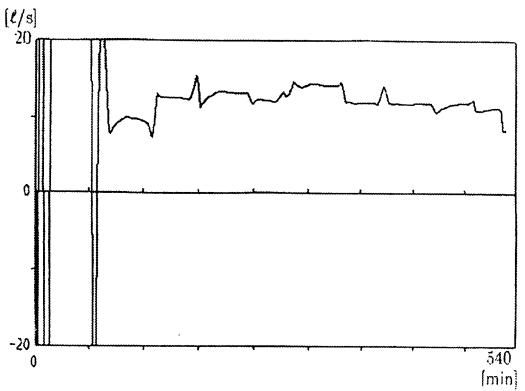


Fig.16.Successive identification airflow rate from chamber-6 to outside.

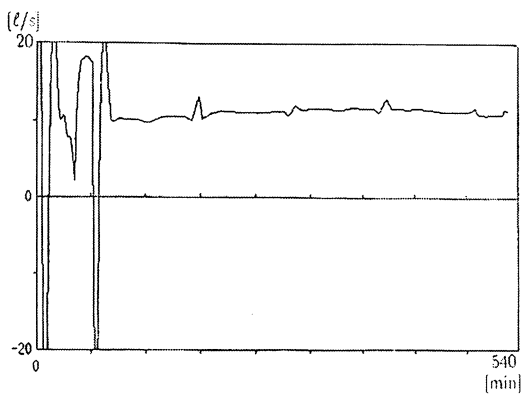


Fig.17.Successive identification airflow rate from outside to chamber-6.

Table 2.Volume of chamber ($m^3$)

| | |
|---|---|
| (1) | 22.29 |
| (2) | 42.37 |
| (3) | 66.12 |
| (4) | 31.86 |
| (5) | 20.79 |
| (6) | 25.45 |
| (7) | 17.14 |
| (8) | 16.43 |
| (9) | 46.15 |

Table 3. Air temperature (°C)

| | |
|---|---|
| (1) | 6.71 |
| (2) | 7.84 |
| (3) | 7.59 |
| (4) | 7.83 |
| (5) | 8.46 |
| (6) | 8.54 |
| (7) | 6.49 |
| (8) | 6.58 |
| (9) | 13.08 |
| (10) | 8.64 |

Table 4. Estimated error of airflow rate in standard deviation ($dm^3/s$)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $\sigma c_{10,9}$ 6.26 | $\sigma c_{10,2}$ 4.77 | $\sigma c_{10,3}$ 4.31 | $\sigma c_{10,4}$ 3.65 | $\sigma c_{10,5}$ 3.40 | $\sigma c_{10,6}$ 4.23 | $\sigma c_{10,7}$ 5.94 | $\sigma c_{10,8}$ 7.01 | $\sigma c_{10,1}$ 4.92 |
| $\sigma c_{2,3}$ 2.40 | $\sigma c_{6,10}$ 1.56 | $\sigma c_{6,7}$ 1.63 | $\sigma c_{5,10}$ 1.07 | $\sigma c_{5,3}$ 1.36 | $\sigma c_{4,10}$ 1.39 | $\sigma c_{4,3}$ 1.54 | $\sigma c_{3,10}$ 2.05 | $\sigma c_{3,7}$ 3.63 |
| $\sigma c_{9,10}$ 2.97 | $\sigma c_{9,8}$ 3.13 | $\sigma c_{8,10}$ 2.85 | $\sigma c_{8,9}$ 5.17 | $\sigma c_{8,7}$ 3.64 | $\sigma c_{8,1}$ 2.74 | $\sigma c_{7,10}$ 1.96 | $\sigma c_{7,8}$ 4.51 | $\sigma c_{7,6}$ 3.75 |
| $\sigma c_{3,5}$ 3.03 | $\sigma c_{3,4}$ 3.09 | $\sigma c_{3,2}$ 3.55 | $\sigma c_{2,10}$ 1.57 | $\sigma c_{2,3}$ 1.46 | $\sigma c_{2,1}$ 2.13 | $\sigma c_{1,10}$ 1.94 | $\sigma c_{1,8}$ 3.46 | $\sigma c_{1,2}$ 2.56 |

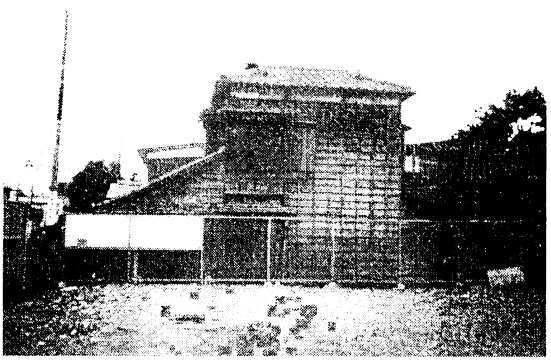

Fig.18. View of experimental house.

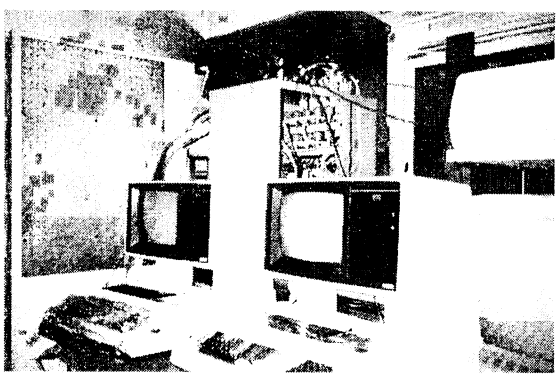

Fig.19. Setting of measurement system in kitchen.

cision. Another problem is the interval of time integration for the system identification. In the present system, the time interval is equal to the measurement interval of one minute. However, it can be supposed that the shorter the interval of time integration for the system identification, the more the identification result is affected by noise, which may have a negative influence on the result. Even if the measurement data has fluctuation due to noise, this can be smoothed by applying the moving average. Consequently, the negative influence of the fluctuation may be dissolved through increasing the time integration interval for the system identification to, say 5 or 10 min.

## 12. CONCLUSION

The tracer gas transfer and diffusion system in the multi-chamber airflow measurement can be modeled by the thermal network. The thermal network model is a general model of the spatially discretized and lumped parameterized transfer and diffusion system. The thermal network model can be mathematically expressed by a general state equation. This state equation is composed of three kinds of parameters $m_{ij}$, $c_{ij}$ and $r_{ij}$. The parameter $c_{ij}$ represents airflow rate in tracer gas transfer and diffusion system, and in case of heat transfer system it means thermal conductance. Two methods of estimating these parameters were introduced on the basis of the least square method. The batch identification method estimates a set of parameters at one time with the total measurement data of state and input variables for a certain length of period. The successive identification method estimates the parameters iteratively with the measurement data obtained at short intervals $\Delta t$. In the latter method, the parameters are simultaneously estimated with the measurement process of state and input values. Moreover, the evaluation method of the errors involved in the estimated results and several indices were introduced, these are based on the residual analysis. Based on this identification theory, a multi-chamber airflow measurement system was developed, and several experiments were conducted with it. By employing the test house in the National Swedish Institute for Building Research, verification experimentation was carried out in the most simple two-room case. As a result, a maximum error of about 20% against the true airflow rate was recognized. The measurement of tracer gas concentration takes much longer time than that of temperature. Therefore, with only a single gas analyser, sequential measurements in a multi-chamber system would involve larger error than for temperature measurement. This is considered to be the major cause of the error. Next, in the experiment on a two-storey wooden house in Tokyo, it was shown that this measurement system can cope with houses of up to nine rooms. The results proved that, the method of using a single kind of tracer gas can deal with a greater number of rooms than methods which use multiple tracer gases. Through these experiments, this system identification theory has been

proven to be feasible. However, concerning the hardware of the measurement system, future improvement is expected.

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