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# Optimizing Theory for State and Energy Supply Based on a Heat and Moisture Transfer Network Model and Numerical Investigation

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## ABSTRACT

A thermal and moisture network model for HVAC systems of buildings, installations and occupants, and an optimizing theory for energy supply and the resulting state are introduced in view of thermal comfort, energy savings and exergy savings. As this method deals with the multi-variable input/output system, it facilitates the design of the optimum thermal comfort distribution for the occupants in a room or optimum energy distribution for complex heating/cooling systems. Numerical investigation proves the advantage of the combination of high thermal insulation, floor heating and natural energy utilization.

## INTRODUCTION

The usual design methods for HVAC systems are not sufficient from the standpoint of optimization for a multi-variable input/output system. When occupants are scattered about in a room, for example, the usual methods have difficulty realizing the optimum thermal comfort distribution. It is also difficult to design the optimum energy distribution for complex heating/cooling systems. Moreover, the usual design methods are insufficient for taking into account the energy quality as indicated by the necessary temperature of the heat transport medium. This insufficiency shrouds the usefulness and proper design of a low exergy/natural energy utilization system. The optimization theory deduced in this paper solves a set of control operation variables and the resulting state based on the linear combination criteria of three factors of thermal comfort, energy savings and exergy savings. This theory is characterized partly by model generality[1]. In the spatial discretization nodal model of an HVAC system including occupants, a parameter called "general conductance" is used to express all heat transfer. Perfectly connected nodal equations, which require no specific thermal/moisture flow balance expressions for all problems, are defined to realize a universally applicable computing program. The following sample investigation deals with the differences between high and low thermal insulation and the differences between a forced convection system, a floor heater and a radiator, while taking into account the location of occupants in the room. This investigation also aims to depict the functional applicability of the theory presented here.

## NETWORK MODEL FOR HEAT AND MOISTURE TRANSFER

The author introduced a heat flow balance equation for a thermal network under the steady state and the  $i$ -th nodal point equation covered by expression (1). For a moisture network, the  $k$ -th nodal point is covered by expression (2). Generalized conductance and free input coefficients have the same meaning as in the previous paper [1]. Subscript  $t$  indicates a heat transfer system, while subscript  $w$  indicates a moisture movement system. Furthermore,  $h_i$  represents thermal flow for evaporation and condensation, while  $j_k$  represents moisture flow for evaporation and condensation.  $n_t$  and  $n_w$  signify the number of dependent nodes (unknown) for temperature and humidity, respectively.  $n_{t0}$  and  $n_{w0}$  indicate the number of independent (given) nodal points for temperature and humidity, respectively.  $n_{tg}$  and  $n_{wg}$  represent the number of free input sources for thermal flow  $g_t$  and  $g_w$ , respectively.

$$\sum_{j=1}^{n+n_{t0}} c_{t_{ij}} \cdot (t_j - t_i) + \sum_{j=1}^{n_{tg}} r_{t_{ij}} \cdot g_{tj} + h_i = 0 \quad (1) \quad \sum_{l=1}^{n+n_{w0}} c_{w_{kl}} \cdot (w_l - w_k) + \sum_{l=1}^{n_{wg}} r_{w_{kl}} \cdot g_{w_{kl}} + j_k = 0 \quad (2)$$

The expressions for thermal and moisture flow on evaporation and condensation can be rewritten as expressions (3) and (4), respectively. In these expressions,  $v$  signifies the product of the mass transfer coefficient, latent heat and area, while  $k$  signifies the product of condensation velocity and area. In expression (3), the  $k$ -th nodal point in the moisture network corresponds to the  $i$ -th nodal point in the thermal network. The  $k$ -th nodal point indicates the saturated vapor pressure in the vicinity of a water film.  $h_i$  is a negative value for evaporation.

$$h_i = \sum_{l=1}^{n_w+n_{w_o}} v_{kl} \cdot (w_l - w_k) \quad (3) \quad j_k = \sum_{l=1}^{n_w+n_{w_o}} k_{kl} \cdot (w_l - w_k) \quad (4)$$

Humidity state independent nodal points  $w_o$  are divided into nodal points for external humidity, and those for saturated humidity in the vicinity of a water film. General formula (5) covers both types of nodal points. For external humidity, coefficients of  $s$  and  $b$  have a value of 0. For saturated humidity, a coefficient of  $w_b$  has a value of 0, and the saturated vapor pressure depends on temperature at the  $i$ -th nodal point, as shown in Wexler-Hyland's equation. Linear approximation at the temperature provides coefficients of  $s$  and  $b$ . First, therefore, the coefficients are determined for assumed water film temperature, followed by optimization described later. Those coefficients are modified at the solution temperature for repeated convergence calculation. Vector  $w_o$  composed of  $n_{w_o}$  humidity independent variables is formulated as expression (6), where  $t$  and  $t_o$  stand for dependent and independent temperature vectors, respectively.

$$w_{o_k} = s_{ki} \cdot t_i + b_k + w_b \quad (5) \quad w_o = S_t \cdot t + S_{t_o} \cdot t_o + b + w_b \quad (6)$$

Here, matrices  $V$  and  $K$  are introduced. They are composed of humidities in vectors  $w_o$ ,  $w$ , and coefficients  $v$  and  $k$  in expressions (3) and (4). Where vector  $w$  comprises dependent (unknown)  $n_w$  nodal humidities. Thus, vector  $h$  ( $n_t$ ) indicating cooling or heating to the thermal transfer system, and vector  $j$  ( $n_w$ ) indicating condensation and evaporation in the moisture movement system are given by expressions (7) and (8), respectively.

$$h = V_w \cdot w + V_{w_o} \cdot w_o \quad (7) \quad j = K_w \cdot w + K_{w_o} \cdot w_o \quad (8)$$

These vectors, and the nodal equations covered in expressions (1) and (2), lead to expressions (9) and (10) representing the steady states of the heat and humidity networks.

$$C_t \cdot t + C_{t_o} \cdot t_o + R_t \cdot g_t + h = 0 \quad (9) \quad C_w \cdot w + C_{w_o} \cdot w_o + R_w \cdot g_w + j = 0 \quad (10)$$

Here, the state vector of size  $(n_t + n_w)$  is defined as  $(t, w)$ , the assigned input vector of size  $(n_{t_o} + n_{w_o})$  as  $(t_o, w_o)$ , and the free input vector of size  $(n_{t_g} + n_{w_g} + n_{w_o})$  as  $(g_t, g_w, b)$ . These definitions allow entirely coupled expression (11) by using (6), (7), (8), (9) and (10).

$$\begin{bmatrix} C_t + V_{wo} \cdot S_t & V_w \\ C_{wo} \cdot S_t + K_{wo} \cdot S_t & C_w + K_w \end{bmatrix} \cdot \begin{bmatrix} t \\ w \end{bmatrix} + \begin{bmatrix} C_{to} + V_{wo} \cdot S_{to} & V_{wo} \\ C_{wo} \cdot S_{to} + K_{wo} \cdot S_{to} & C_{wo} + K_{wo} \end{bmatrix} \cdot \begin{bmatrix} t_o \\ w_b \end{bmatrix} + \begin{bmatrix} R_t & 0 & V_{wo} \\ 0 & R_w & C_{wo} + K_{wo} \end{bmatrix} \cdot \begin{bmatrix} g_t \\ g_w \\ b \end{bmatrix} = 0 \quad (11)$$

If the state vector is represented by  $\mathbf{x}$ , the assigned input vector by  $\mathbf{x}_o$ , and the free input vector by  $\mathbf{g}$ , whole equation (11) is simplified to the expression (12).

$$\mathbf{C} \cdot \mathbf{x} + \mathbf{C}_o \cdot \mathbf{x}_o + \mathbf{R} \cdot \mathbf{g} = \mathbf{0} \quad (12)$$

Now, driving variables must be rearranged to introduce control vector  $\mathbf{u}_c$  and disturbance vector  $\mathbf{u}_d$ . Therefore, the elements in vectors are given specified order. Input vector  $\mathbf{u}_{tc}$  on the heat transfer system precedes input vector  $\mathbf{u}_{wc}$  on the moisture movement system, with assigned input elements followed by free input elements. Moreover, the numbers of nodal points or sources performing control precede the numbers of nodal points and sources that act as disturbance. These conventions enable  $\mathbf{u}_c$  and  $\mathbf{u}_d$  to be defined as expressions (13) and (14), respectively. A subscript of c indicates vector elements originating in control, while a subscript of d indicates vector elements stemming from disturbance. A t located at the top left indicates matrix transpose.

$$\mathbf{u}_c = {}^t(\mathbf{u}_{tc}, \mathbf{u}_{wc}) = {}^t({}^t\mathbf{t}_{oc}, {}^t\mathbf{g}_{tc}, {}^t\mathbf{w}_{bc}, {}^t\mathbf{g}_{wc}) \quad (13) \quad \mathbf{u}_d = {}^t({}^t\mathbf{t}_{od}, {}^t\mathbf{g}_{td}, {}^t\mathbf{w}_{bd}, {}^t\mathbf{g}_{wd}, {}^t\mathbf{b}) \quad (14)$$

The next step is to draw column vectors out of the drive matrices  $\mathbf{C}_o$  and  $\mathbf{R}$  according to the internal configuration of vector elements resulting from control and disturbance. Now, we get expression (15) for a created drive matrix for control  $\mathbf{D}_c$  and disturbance  $\mathbf{D}_d$ .

$$\mathbf{C} \cdot \mathbf{x} + \mathbf{D}_c \cdot \mathbf{u}_c + \mathbf{D}_d \cdot \mathbf{u}_d = \mathbf{0} \quad (15)$$

The size of the state vector has been enlarged because of coupling with the moisture network.

## OPTIMIZATION

The goal for optimum control is to keep the central temperature of occupants at 36.8 degrees C and to set air humidity in a room at a certain value. These controls and settings must be implemented so that the sum of the square of the deviations from 0 or from environment temperature are minimized on two or more control variables. The criteria for this implementation is given by expression (16). The first term is concerned with the control object and the second term with control variables.  $\mathbf{t}_r$  and  $\mathbf{w}_r$  provide control object values for temperature and humidity, respectively, while  $\mathbf{d}_t$  and  $\mathbf{d}_w$  provide control reference values for the heat transfer and moisture movement systems, respectively. Environment temperature is defined by  $\mathbf{d}_t$  and  $\mathbf{d}_w$ . Various types of weighing matrices  $\mathbf{W}$  in the quadratic form are composed mainly of diagonal elements.  $\mathbf{W}_t$  stands for the heat transfer system, while  $\mathbf{W}_w$

stands for the moisture movement system. A subscript of c indicates that the matrix is concerned with control variables. Expression (16) can be rewritten as concise expression (17). In  $\mathbf{W}_x$ , only the diagonal elements corresponding to state vector elements subjected to control are not zero. In  $\mathbf{W}_c$ , all diagonal elements are not zero (positive definite).

$$J = \begin{bmatrix} t \\ w \end{bmatrix} - \begin{bmatrix} t_r \\ w_r \end{bmatrix} \cdot \begin{bmatrix} W_t & 0 \\ 0 & W_w \end{bmatrix} \cdot \begin{bmatrix} t \\ w \end{bmatrix} - \begin{bmatrix} t_r \\ w_r \end{bmatrix} + \begin{bmatrix} u_{tc} \\ u_{wc} \end{bmatrix} - \begin{bmatrix} d_t \\ d_w \end{bmatrix} \cdot \begin{bmatrix} W_{tc} & 0 \\ 0 & W_{wc} \end{bmatrix} \cdot \begin{bmatrix} u_{tc} \\ u_{wc} \end{bmatrix} - \begin{bmatrix} d_t \\ d_w \end{bmatrix} \quad (16)$$

$$J = {}^t(x-r) \cdot W_x \cdot (x-r) + {}^t(u_c-d) \cdot W_c \cdot (u_c-d) \quad (17)$$

Where,  $\mathbf{r}$  represents  $(t_r, w_r)$ , and  $\mathbf{d}$  represents  $(d_t, d_w)$ . The solution of  $\mathbf{x}$  in expression (15) is substituted into this criteria. The resulting expression is differentiated by control vector  $\mathbf{u}_c$  and is equated to  $\mathbf{0}$ . Hence,  $\mathbf{u}_c^*$  for optimizing the criteria is solved in equation (18).

$$u_c^* = -[{}^tD_c \cdot {}^tC^{-1} \cdot W_x \cdot C^{-1} \cdot D_c + W_c]^{-1} \cdot [{}^tD_c \cdot {}^tC^{-1} \cdot W_x \cdot C^{-1} \cdot D_d \cdot u_d + {}^tD_c \cdot {}^tC^{-1} \cdot W_x \cdot r - W_c \cdot d] \quad (18)$$

Substituting the  $\mathbf{u}_c^*$  into expression (15) in regard to  $\mathbf{x}$  provides optimum state  $\mathbf{x}^*$ . The nonlinearity of dependence of thermal radiation, convective transfer and other factors on temperature can be covered by repeated convergence calculation. On the basis of the above theory, optimization calculation program SOCS was created.

## HVAC SYSTEM MODEL

Figure 1 illustrates the building considered. It had a width of 3.665 meters, a ceiling height of 2.45 meters, a depth of 4.55 meters, a floor area of 16.68 square meters, and a room volume of 40.86 cubic meters. The window had a longitudinal length of 1.5 meters, and a lateral length of 2.55 meters. The panel heater had a longitudinal length of 0.35 meters, and a lateral length of 1.61 meters. The wall, the roof, the ceiling, and the floor were provided with glass wool 35K offering thermal insulation thicknesses of 140, 190, 210 and 235 mm, respectively. The low thermal insulation model had a thickness of 50 mm. The interior is provided with humidity-proof layers. The forced convection system provided a hot water flow rate of 2.5  $\ell/m$ , and an air flow rate of 275.4 cubic meters per hour. The heat exchanger efficiency of total heat exchangers [2] is 0.69, enthalpy efficiency was 0.6, and the rate of ventilation was 90 cubic meters per hour. The hot water flow rate for the floor and panel heaters was 2.5  $\ell/m$ . In this study, the outside temperature was assumed to be 0 degrees C. The room width was divided equally by six, with each of three occupants located at one of three equal partitions. The metabolism was assumed to be 1.5 met, though it may be somewhat high, under 0.8 clo. Ambient conditions were such that the water vapor pressure was 11.2 mmHg and the air velocity was 0.1 m/s. Figure 2 illustrates the network model for heat transfer. Discretization for the system was a finite volume, the same as in the previous paper [1], and generalized conductances were obtained. The equipments were modeled on the basis of catalog values[2][3]. Thermal loss of the piping was modeled on the assumption that it was insulated with 20 mm of rock wool and its total length was 10 meters. Form factors of radiation heat transfer related to the occupants were determined according to conventional method[4] as if they were cubes. The heat transfer model of a occupant has been proven by PMV. The dependence of convective transfer from a occupant on the temperature, and the nonlinearity of the dependence of radiation on the temperature were covered by convergence calculation.

## OPTIMIZATION RESULT

This optimization theory is concerned with optimum control. The control variable is the central temperature of three occupants, with the target value being 36.85 degrees C. The operation variable is heat flow input to hot water in heating systems. For humidity, the control variable is room humidity, while the operation variable is humidifying the room air. Input heat flow is weighted by  $10^2$ , humidifying by 10, bodily temperature deviations by  $10^6$ , and humidity deviations by  $10^9$ . The heating systems examined for comparison are a floor heating system, a forced air convection system, a panel heating system, and a combination of these systems. Tables 1 and 2 show optimization calculations for input heat flow, inlet hot water temperature, air temperature, and MRT in regard to high and low thermal insulation. On both high and low thermal insulation, the floor heater, the panel heater, and the forced convection system are characterized by less input heat flow in that order. Floor heating based on high thermal insulation can be satisfied with a very low temperature hot water of 29.96 degrees C. Therefore, this heating system is well adapted to using low-quality natural energy such as solar heat. Differences resulting from the location of occupants, which are judged by realized body temperature, vary with thermal radiation on the window surface and the panel heater. If the heat flow into the occupant is adopted as a control operation and weighted more significantly, the heat flow may be related to the thermal sensation.

## CONCLUSION

This theory can be applied to a total system covering building, equipment and occupants. It enables optimum design in view of thermal comfort, energy savings and exergy savings. Hence, the optimum heat medium temperature can be designed in consideration of exergy savings. Moreover, as this theory provides optimization for a multi-variable I/O system, it is also applicable to a complex heating/cooling system with two or more operation variables to determine optimum energy distribution. In addition, two or more occupants and control variables can be covered. This sample investigation has proven that a combination of high thermal insulation, floor heating and natural energy utilization is desirable from the standpoint of energy and exergy savings.

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## REFERENCES

- [1]Okuyama, H. 1987 Doctorate thesis,Theoretical Study on the Thermal Network Model in Buildings
- [2]Matsushita Electric Industrial Co.,Ltd. 1993 Technical manual TM-21, Heating system with heat recovery for high thermal insulation and airtight house, pp6-7
- [3]Sunpot Co., Ltd. 1994,Technical manual 94A, Hot water heating system,p.14,p.81,p. 83,pp44-52
- [4]Shukuya, M. 1993,Light and Heat in the Built Environment, Maruzen Co., Ltd. ISBN4-621-03862-1 C3052

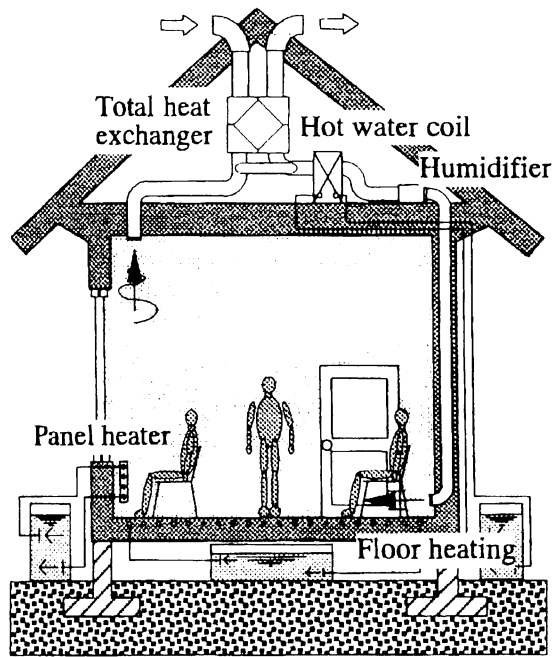


Figure 1 HVAC system

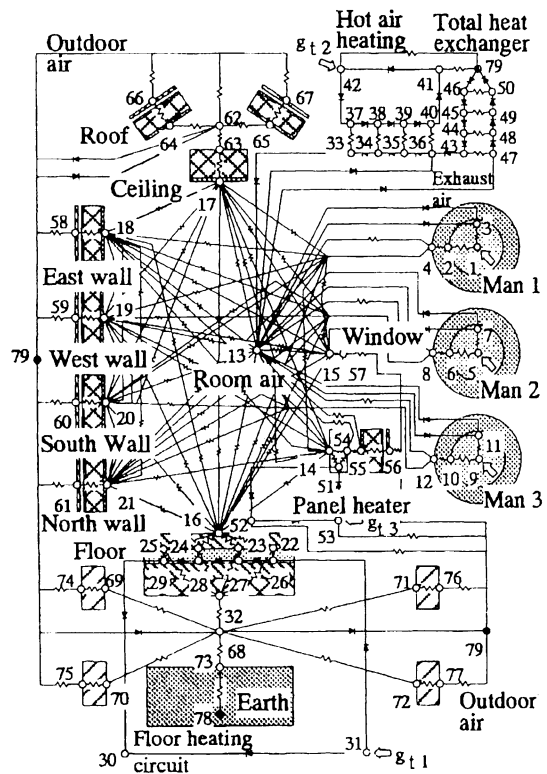


Figure 2 Thermal network model

Table 1 Optimum result in high thermal insulation

		Floor heating	Hot air heating	Panel heating	Combined heating
Energy supply for water circuit(kW)		0.7071	0.8261	0.7991	0.7777
Water inlet temperature(°C)		29.96	43.11	48.38	—
Atmospheric air temperature(°C)		17.24	17.95	17.65	17.60
Mean Radiant Temperature MRT(°C)	Man 1	17.79	16.34	17.62	17.38
	Man 2	18.33	17.03	17.25	17.52
	Man 3	18.57	17.10	17.19	17.58
Realized body center temperature(°C)	Man 1	36.73	36.72	36.94	36.81
	Man 2	36.86	36.89	36.80	36.84
	Man 3	36.95	36.93	36.80	36.87

Table 2 Optimum result in low thermal insulation

		Floor heating	Hot air heating	Panel heating	Combined heating
Energy supply for water circuit(kW)		1.459	1.566	1.502	1.509
Water inlet temperature(°C)		41.06	76.76	77.02	—
Atmospheric air temperature(°C)		17.18	18.55	17.90	17.89
Mean Radiant Temperature MRT(°C)	Man 1	17.42	14.70	17.09	16.58
	Man 2	18.38	15.93	16.34	16.84
	Man 3	18.90	16.12	16.28	17.02
Realized body center temperature(°C)	Man 1	36.65	36.63	37.08	36.81
	Man 2	36.86	36.92	36.73	36.82
	Man 3	37.02	36.73	36.71	36.88