Least squares solution of $\boxed{a}$ (vector) including system identification coefficients $c_{i,j}$, $m_{i,j}$ and $r_{i,j}$

Please see details for ref. [99],2012

State space equation,
Simultaneous ordinary differential equations for state space vector **x**

$$\mathbf{M}\cdot\dot{\mathbf{x}} = \mathbf{C}\cdot\mathbf{x} + \mathbf{C_o}\cdot\mathbf{x_o} + \mathbf{R}\cdot\mathbf{g} \quad (2)$$

Regression equation for vector **a**

$$\mathbf{y}_k = \mathbf{Z}_k\cdot\boxed{\mathbf{a}} \quad (3)$$

Least squares from state space equation

$$\left[\boxed{\sum_{k=1}^{nt} {}^t\mathbf{Z}_k\cdot\mathbf{Z}_k}\right]\cdot\mathbf{a} = \boxed{\sum_{k=1}^{nt} {}^t\mathbf{Z}_k\cdot\mathbf{y}_k} \quad (4)$$

Constraints of **a**

$$\mathbf{S}\cdot\boxed{\mathbf{a}} = \mathbf{d}$$

Least squares from constraints

$$\boxed{{}^t\mathbf{S}\cdot\mathbf{S}}\cdot\mathbf{a} = \boxed{{}^t\mathbf{S}\cdot\mathbf{d}} \quad (5)$$

$$\mathbf{F} = \begin{bmatrix} \sum_{k=1}^{nt} {}^t\mathbf{Z}_k\cdot\mathbf{Z}_k \\ {}^t\mathbf{S}\cdot\mathbf{S} \end{bmatrix} \quad (6) \qquad \mathbf{b} = \begin{bmatrix} \sum_{k=1}^{nt} {}^t\mathbf{Z}_k\cdot\mathbf{y}_k \\ {}^t\mathbf{S}\cdot\mathbf{d} \end{bmatrix} \quad (7)$$

Least squares solution of **a**

$$J = {}^t(\mathbf{b}-\mathbf{F}\cdot\mathbf{a})\cdot\mathbf{W}\cdot(\mathbf{b}-\mathbf{F}\cdot\mathbf{a}) \quad (8)$$

$$\hat{\mathbf{a}} = \left({}^t\mathbf{F}\cdot\mathbf{W}\cdot\mathbf{F}\right)^{-1}\cdot\left({}^t\mathbf{F}\cdot\mathbf{W}\cdot\mathbf{b}\right) \quad (9)$$

(Here **W** is a weighting matrix
that makes the maximum value of each row to 1)

A system of ordinary differential equations is constructed from the nodal equations, and a regression equation for **a** is constructed by transforming this. This yields the first-stage least squares solution.
On the other hand, the first-stage least squares solution can also be obtained from the constraint expressions such as the symmetry of $c_{i,j}=c_{j,i}$.
By arranging these two equations in the direction of increasing rows, the second-stage least squares solution equation is derived.
Therefore, a double least squares method is applied.

The deduction of equation (5) will be explained in a later slide.