Constraint equations for the system identification coefficients and contribution equations to the least squares method

Here explain a vector **a** containing coefficients to be identified and constraint equations of these coefficients.

An example of contents of the matrix **S**, vectors **d** and **a** are as following (13), (14) and (15).

$$\mathbf{S} = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & -1 & 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix} \quad (13), \quad \mathbf{d} = \begin{pmatrix} c_p \cdot \rho \cdot (q_{i,j} - q_{j,i}) \\ 0 \\ \sum_{k=1}^{n+no} c_{i,k} - \sum_{k=1}^{n+no} c_{k,i} = 0 \\ \cdot \end{pmatrix} \quad (14), \quad \mathbf{a} = \begin{pmatrix} c_{i,j} \\ c_{j,i} \\ c_{k,i} \\ c_{i,k} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{pmatrix} \quad (15)$$

In this example, the first row shows previous slides eq.(12), second row shows symmetry of $c_{k,i}=c_{i,k}$, third row shows the flow balance of the generalized conductance concerning node i .

These can be expressed as a simple matrix and vector equation (16).

$$\mathbf{S} \cdot \mathbf{a} = \mathbf{d} \quad (16)$$

For the application of least squares we define a error vector as (17).

$$\mathbf{e} = \mathbf{S} \cdot \mathbf{a} - \mathbf{d} \quad (17)$$

An evaluating function Jc of the least squares method is written by equation (18).

$$J_c = {}^t\mathbf{e} \cdot \mathbf{e} = \left({}^t\mathbf{a} \cdot {}^t\mathbf{S} - {}^t\mathbf{d}\right)\left(\mathbf{S} \cdot \mathbf{a} - \mathbf{d}\right) = {}^t\mathbf{a} \cdot {}^t\mathbf{S} \cdot \mathbf{S} \cdot \mathbf{a} - {}^t\mathbf{a} \cdot {}^t\mathbf{S} \cdot \mathbf{d} - {}^t\mathbf{d} \cdot \mathbf{S} \cdot \mathbf{a} + {}^t\mathbf{d} \cdot \mathbf{d} \quad (18)$$

By differentiating the evaluation function Jc with respect to vector **a**, we obtain equation (19).

$$\frac{\partial J_c}{\partial \mathbf{a}} = 2 \cdot {}^t\mathbf{S} \cdot \mathbf{S} \cdot \mathbf{a} - 2 \cdot {}^t\mathbf{S} \cdot \mathbf{d} = 0 \quad (19)$$

The constraint equation that contributes to half of the total least squares is obtained as equation (5) from equation (19).

$${}^t\mathbf{S} \cdot \mathbf{S} \cdot \mathbf{a} = {}^t\mathbf{S} \cdot \mathbf{d} \quad (5)$$