

Flow rate balances to be used in multizone air movement network models of buildings

The absolute temperature at 0 degrees Celsius [$^{\circ}\text{C}$] is $T_0 = 273.16$ [K], and the air density is $\rho_0 = 1.293$ [kg/m^3]. The air density at absolute temperature T [K] is defined as ρ [kg/m^3]. Then, according to Gay-Lussac's law, $\rho = (T_0/T)\rho_0$, and by setting $d_m = 353.2$, we can write the calculation formula (1) for ρ .

$$\rho = (1.293 \cdot 273.16) / T = 353.2 / T = d_m / T \quad (1)$$

Considering only heat transfer due to air movement and assuming a steady state, we can write heat flow balance equation (2) where air with absolute temperature T_j and density ρ_j flows in from zone j to zone i at volumetric flow rate $q_{i,j}$, and air with temperature T_i and density ρ_i flows out from zone i at flow rate $q_{j,i}$. The specific heat at constant pressure c_p is approximately constant at 1005 [$\text{J}/\text{kg} \cdot \text{K}$]. Using the above equation (1), the temperature variable can be eliminated, leading to equation (3) which shows the volume flow rate balance. This is the flow rate balance equation that must be satisfied in the air flow network model.

$$\begin{aligned} & \sum_{j=1} c_{i,j} \cdot T_j - \sum_{j=1} c_{j,i} \cdot T_i \\ &= \sum_{j=1} c_p \cdot \rho_j \cdot q_{i,j} \cdot T_j - \sum_{j=1} c_p \cdot \rho_i \cdot q_{j,i} \cdot T_i \\ &= \sum_{j=1} c_p \cdot \frac{d_m}{T_j} \cdot q_{i,j} \cdot T_j - \sum_{j=1} c_p \cdot \frac{d_m}{T_i} \cdot q_{j,i} \cdot T_i \\ &= c_p \cdot d_m \cdot \left(\sum_{j=1} q_{i,j} - \sum_{j=1} q_{j,i} \right) = 0 \end{aligned} \quad (2)$$

$$\sum_{j=1} q_{i,j} - \sum_{j=1} q_{j,i} = 0 \quad (3)$$

